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# Presolve analysis and interior point solutions of the linear programming coordination problem of directional overcurrent relays

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## Abstract

A linear programming interior point algorithm is proposed for the solution of the problem of coordinating directional overcurrent relays in interconnected power systems considering definite time backup relaying. The proposed algorithm is a variation of the primal–dual approach that uses multiple correctors of centrality. Pre-solution problem filtering simplification techniques are used prior to the application of the linear programming algorithm. Results are presented for the application of the methodology on a realistic test case, a 115–69 kV power system with 108 buses, 86 lines, 61 transformers, and 97 directional overcurrent relays. Optimal solutions are found in an automatic fashion, using the algorithm for the settings of the ground relays as well as for the phase relays. The application of the pre-solution problem simplification techniques is highly recommended, resulting in a significant reduction of the size and complexity of the linear programming problem to be solved. The interior point approach reaches a feasible point in the close vicinity of the final optimal result in only one or two iterations. This fact represents an advantage for on-line applications. The proposed methodology and in particular the use of the presolve problem simplification techniques is shown as a new valuable tool for the setting of directional overcurrent relays in interconnected power systems. 

*Keywords*: Power system protection; Relay coordination; Linear programming; Interior point algorithms

## 1. Introduction

The determination of the time dial settings of directional overcurrent relays in meshed power systems in order to comply with the requirements of sensibility, selectivity, reliability and speed was stated as an optimization problem in 1987 [1]. The application of the simplex method for linear programming to the solution of this problem has been successful [1–4,8].

The problem of re-calculating the settings of the relays after a network expansion, reducing the number of relays to be reset, was solved by means of an iterative application of the optimization methodology proposed in Refs. [1–8]. In this case, the problem is augmented in each iteration and different solutions are found successively using the linear programming technique. Multiobjective optimization concepts were applied to find a tradeoff between the number of relays to be reset and the sum of the operation times.

The consideration of the transient changes of the system configuration that take place during the fault clearing process was also recently addressed in Ref. [4].

The consideration of definite time relaying, i.e. instantaneous units, distance relays and breaker failure relays for the formulation and solution of the optimization problem is presented in Ref. [5] and a comparison of the application of the feasibility of the relay time coordination under different criteria is studied in Ref. [9].

However, up to date, the optimal relay coordination problem has been solved only using the traditional simplex method for linear programming.

The purpose of this work is to evaluate the goodness of the proposed pre-solution techniques when applied to this particular problem, as well as the application of a different optimization technique, a primal–dual interior point predictor–corrector approach with multiple correctors of centrality.

## 2. Statement of the problem

The coordination of the operation of directional overcurrent relays in interconnected power systems can be stated as an optimization problem in the following fashion [1,8]:

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Minimize: weighted sum of operation times
Subject to:

relay time–current characteristics,
relay operation time coordination constraints that assure the selectivity of the solution:
- associated to directional overcurrent relays
- associated to definite time backup relaying:
  - instantaneous units
    - distance relays
  - breaker failure relays
- physical limits on the relay settings,
- limits on the operation times.

The relay time versus current characteristics are given by the equations for the inverse-type overcurrent relays that express the operation time as a function of the pickup current multiple. These equations are a product of two factors: $x_i = g_i(D_i)$ that depend on the time dial settings $D_i$ and $f_i(M_i(Fp))$ functions of the pick-up current settings, where $M_i(Fp)$ is the multiple of the pickup current for relay $i$ for fault $Fp$ [1,8].

For the purpose of the problem treated hereon, the relay pick-up currents and the current transformer ratios are assumed to be previously defined by the engineers and considered as known parameters; therefore, the relay time–current characteristics can be expressed as linear expressions of the operation times in terms of the variables $x_i = g_i(D_i)$.

It has been demonstrated that the optimal solution is independent on the selection of the weighting factors and therefore they are all set equal to one. Consequently, the objective function can be defined as the sum of the time dial settings of all relays of the system leading to the minimum relay operation times [1].

The selectivity constraints are included to enforce a time difference, called coordination time interval in the operation times of the main and backup relays, for a predetermined set of fault conditions. These equations are linear expressions in terms of the operation times for the "close-in" and "remote" fault conditions, for each main-backup relay pair. They depend on the operation times of the directional overcurrent relays and of the definite time units, the latter ones considered as known parameters for the solution of the problem treated hereon [8,9]. In general, these expressions evaluate the operation times of the main-backup relay pairs, for two given fault conditions, which represent the limits of the perturbation space. In Ref. [8], a more detailed presentation is given about the consideration of definite time relaying and the cases of instantaneous units, distance relays and breaker failure relays are treated separately.

Other constraints like the maximum and minimum permissible values for the time dial settings or in the operation times are treated in the same way.

The optimization problem, therefore, can be stated as a linear programming problem in the standard form, as follows:

$$\text{minimize } c^{\mathrm{T}}x = \sum_{i=1}^{r} c_i x_i \tag{1}$$

$$\text{subject to : } Ax \leq b. \tag{2}$$

It must be pointed out that the desired relay settings $D_i$ are calculated once the solution of Eqs. (1) and (2) has been found, by solving the non-linear equations that represent the relay characteristics:

$$g_i(D_i) - x_i = 0, \qquad i = 1, 2, \ldots, r \tag{3}$$

The settings are treated as continuous variables. It is understood that they have to be fixed according to the resolution allowed by the relays being used.

## 3. Solution methodology

In order to solve the linear programming problem stated in Eqs. (1) and (2), an interior point primal–dual algorithm developed by Gondzio [10] was used. This method, called higher-order primal dual method (HOPDM), is a variant of the original primal–dual algorithm first introduced by Kojima et al. [11], based on the work by Megiddo [12]. A detailed study about primal–dual interior point methods can be found in Wright [13]. These methods have been shown to be very successful computationally. Indeed, they are competitive with state-of-the-art implementations of the simplex method, especially for large-scale problems (see Lustig et al. [14], Mehrotra [15] and Gondzio and Terlaky [16] for a survey on computational issues). HOPDM uses a strategy involving multiple correctors of centrality, which appears to be of further computational advantage.

In this work, we used version 2.11 of HOPDM, which is freely available through Netlib for academic use in Fortran 77 source code.

The algorithm first transforms problems (1) and (2) into a more suitable form for the interior point method:

$$\begin{aligned} &\text{minimize} \quad c^{\mathrm{T}}x \\ &\text{subject to} \quad Ax = b, \ \ 0 \leq x \leq u, \end{aligned} \tag{4}$$

where $c, x, u \in \Re^n$, $b \in \Re^m$, $A \in \Re^{m\times n}$.

### 3.1. The presolve phase

Next, a presolve phase is performed in order to simplify the formulation and make it easier to solve. It is worth noting that presolving is regarded as a very effective technique in linear programming, not only for interior point methods but also for simplex-type algorithms [17]. In particular, the presolve implemented in HOPDM is considered state-of-the-art and is customized for the interior point method used.

Problem size reduction is achieved by analyzing the rows,