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On duality in linear programming under fuzzy environment

C.R. Bector[a,*], S. Chandra[b]

[a]Department of Business Administration, University of Manitoba, Winnipeg, Man., Canada R3T 5V4

[b]Department of Mathematics, Indian Institute of Technology, Hauz Khas, New Delhi 110016, India

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Abstract

A pair of linear primal–dual problems is introduced under fuzzy environment and appropriate results are proved to establish the duality relationship between them. Possible extensions are also suggested. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

A number of researchers have exhibited their interest in the topic of fuzzy linear programming after Zadeh [5] developed the concept of fuzzy set theory. However, in contrast with the vast literature on modeling and solution procedures for a linear program in a fuzzy environment (see, for example, [3,7]), the studies in duality are rather scarce. The most basic results on duality in fuzzy linear programming are due to Rodder and Zimmermann [4] and Hamacher et al. [2]. In [4] a generalization of maxmin and minmax problems in a fuzzy environment is presented and thereby a pair of fuzzy dual linear programming problems is constructed. An economic interpretation of this duality in terms of market and industry is also included in [4]. The paper by Hamacher et al. [2] is mostly devoted to the study of sensitivity analysis in fuzzy linear programming.

The purpose of the present paper is twofold. Firstly, to observe that there are certain inherent difficulties with the fuzzy dual formulations of [4] because when they are specialized to the crisp situation they do not lead to a standard primal–dual pair for linear programming, and secondly, to construct a modified pair of fuzzy primal–dual linear programming problems. To achieve this, we divide the paper into three sections. In Section 1, we introduce the fuzzy dual formulation of Rodder and Zimmermann [4] and make certain observations on their formulation. In Section 2 a modified formulation is presented, whereas its comparison and possible extensions are included in Section 3.

* Corresponding author. Tel.: +1-204-474-8445; fax: +1-204-474-7545.
E-mail address: bector@ms.umanitoba.ca (C.R. Bector).

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2. Rodder–Zimmermann formulation of a fuzzy linear program dual

Let R^n denote the n-dimensional Euclidean space and R^n_+ be its non-negative orthant. Let $c \in R^n$, $x \in R^n$, $b \in R^m$, $w \in R^m$, and the matrix $A \in R^m \times R^n$. It is well known that for the following crisp primal–dual linear programming pair:

(P) maximize $c^T x$ subject to $Ax \leqslant b, \quad x \geqslant 0$ and

(D) minimize $b^T w$ subject to $A^T w \geqslant c, \quad w \geqslant 0$

a sound and fascinating duality theory is available. This theory has numerous business, economic, and industrial interpretations and applications. In particular, we can identify problems (P) and (D) as the industry and the market problems, respectively. In case of fuzzy environment, neither the industry nor the market is a strict maximizer or minimizer but are rather satisfier. Taking $c^T x^0$ and $b^T w^0$ as the aspiration levels of the industry (I) and the market (M), respectively, Rodder and Zimmermann [4], introduced the following membership functions:

$$\mu^I(x) = \begin{cases} 1 & \text{if } c^T x^0 \leqslant c^T x, \\ 1 - (c^T x^0 - c^T x) & \text{otherwise,} \end{cases}$$

$$\mu^I_x(w) = \begin{cases} 0 & \text{if } w^T(b - Ax) \leqslant 0, \\ w^T(b - Ax) & \text{otherwise,} \end{cases}$$

$$\mu^M(w) = \begin{cases} 1 & \text{if } b^T w \leqslant b^T w^0, \\ 1 - (b^T w - b^T w^0) & \text{otherwise,} \end{cases}$$

$$\mu^M_w(x) = \begin{cases} 0 & \text{if } x^T(c - A^T w) \geqslant 0, \\ -x^T(c - A^T w) & \text{otherwise} \end{cases}$$

and presented their economic interpretations. Using the above membership functions, they constructed the following pair of problems and called them fuzzy pair of primal–dual problems.

$(\mathrm{FP})_w$ maximize λ_1

subject to
$$\lambda_1 \leqslant 1 - (c^T x^0 - c^T x), \tag{1}$$
$$\lambda_1 \leqslant w^T(b - Ax) \quad \text{(for any given } w \geqslant 0), \tag{2}$$
$$x \in R^n_+, \quad \lambda_1 \in R, \tag{3}$$

$(\mathrm{FD})_x$ minimize λ_2

subject to
$$\lambda_2 \geqslant (b^T w - b^T w^0) - 1, \tag{4}$$
$$\lambda_2 \geqslant x^T(c - A^T w) \quad \text{(for any given } x \geqslant 0), \tag{5}$$
$$w \in R^m_+, \quad \lambda_2 \in R. \tag{6}$$

It may be noted here that there are numerous problems of type (FP) (respectively, (FD)) for any given $w \geqslant 0$ (respectively, $x \geqslant 0$). Therefore, to emphasize this point and make it more explicit, we have denoted the above problems as $(\mathrm{FP})_w$ and $(\mathrm{FD})_x$ and not, as is done in [4], by (FP) and (FD).

Apart from this construction of the fuzzy dual, an important result of [4] is the fuzzy equivalent version of the usual weak duality theorem for the pair (FP), (FD).