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Interactive fuzzy programming for decentralized two-level linear programming problems

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Abstract

This paper presents interactive fuzzy programming for a decentralized two-level linear programming problem with a single decision maker (DM) at the upper level and multiple DMs at the lower level. The interactive method consists of two phases. In the first phase, after determining the fuzzy goals of the DMs at both levels, the DM at the upper level subjectively specifies a minimal satisfactory level. Taking into consideration overall satisfactory balance between the two levels, the DM at the upper level updates the minimal satisfactory level if necessary, and a tentative solution is obtained. In the second phase, consulting the ratios of satisfaction between the DM at the upper level and each of the DMs at the lower level, the DM at the upper level specifies maximal satisfactory levels to the DMs at the lower level and updates them if necessary. A satisfactory solution can be derived efficiently through the interactive procedure of the two phases. A numerical example illustrates the proposed method.

Keywords: Decentralized two-level linear programming problem; Interactive fuzzy programming; Fuzzy goals

1. Introduction

In this paper, we consider two-level programming problems in which there are a single decision maker (DM) at the upper level and two or more DMs at the lower level, and objective functions of the DMs and constraint functions are linear functions. For such decentralized two-level linear programming problems, Simaan and Cruz [9] and Anandalingam [1] supposed that the DMs at the lower level make decisions so as to equilibrate their objective function values for a decision of the DM at the upper level on condition that all the DMs at the lower level do not have motivation to cooperate mutually. Namely, they supposed that the DM at the upper level assumes that rational reactions of the DMs at the lower level with respect to the decision of

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the DM at the upper level are a Nash equilibrium solution and the DM at the upper level selects a decision which optimizes his or her objective function. In this paper, we call such solutions Stackelberg solutions.

First of all, consider a two-level problem with a single DM at each level. When the Stackelberg solution is employed, it is assumed that there is no communication between the two DMs, or they do not make any binding agreement even if there exists such communication. The above assumption, however, is not always reasonable when we model decision making problems in a decentralized firm as a two-level programming problem in which top management is the DM at the upper level and an operation division of the firm is the DM at the lower level because it is supposed that there exists a cooperative relationship between them or a relationship of the principal and accessory.

Consider a computational aspect of the Stackelberg solution. Even if objective functions of both DMs and common constraint functions are linear, it is known that the problem for obtaining the Stackelberg solution is a non-convex programming problem with special structure. Although a large number of algorithms for obtaining the Stackelberg solution have been developed [10], it is known that the problem finding the solution is strongly NP-hard [8]. From such difficulties, a new solution concept which is easy to compute and reflects the structure of two-level programming problems is expected.

Recently, Lai [3] and Shih, Lai and Lee [7] proposed a solution concept, which was different from the concept of the Stackelberg solution, for two-level or multi-level linear programming problems such that decisions of DMs in both levels are sequential and all of the DMs essentially cooperate with each other.

Their method is based on the idea that the DM at the lower level optimizes his or her objective function, taking a goal or preference of the DM at the upper level into consideration. The DMs identify membership functions of fuzzy goals for their objective functions, and especially, the DM at the upper level also specifies those of fuzzy goals for decision variables. The DM at the lower level solves a fuzzy programming problem with constraints on fuzzy goals of the DM at the upper level. Unfortunately, however, there is a possibility that their method leads a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and the decision variables.

To overcome the problem in the methods of Lai et al., eliminating the fuzzy goals for the decision variables, Sakawa et al. have developed interactive fuzzy programming for two-level linear programming problems [5]. In this paper, we present interactive fuzzy programming for the decentralized two-level linear programming problems with a single DM at the upper level and multiple DMs at the lower level.

The proposed method consists of two phases. In the first phase, the DMs at both levels identify membership functions for their fuzzy goals for the objective functions. Taking into consideration overall satisfactory balance between the two levels, the DM at the upper level specifies the minimal satisfactory level and updates it if necessary, and then a tentative solution is obtained. In this phase, the DMs at the lower level are treated impartially and therefore, they can be regarded as a group.

In the second phase, consulting the ratios of satisfaction between the DM at the upper level and each of the DMs at the lower level, the DM at the upper level specifies maximal satisfactory levels to some of the DMs at the lower level and updates them if necessary. By coordinating the satisfaction degrees of the DMs, the final satisfactory solution can be derived.

In this interactive process, although more attention or respect to the DM at the upper level is paid, the finally obtained satisfactory solution becomes a well-balanced solution not only between both the levels but also between the DM at the upper level and each of the DMs at the lower level. Finally, a numerical example is given to illustrate the proposed method.

2. Formulation and example

In this paper, we consider a situation where there are a single decision maker (DM0) at the upper level and k decision makers (DM1, ..., DMk) at the lower level. We employ the following representation in order