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# Moments of multivariate regime switching with application to risk-return trade-off ☆

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ABSTRACT

We use a Fourier transform to derive multivariate conditional and unconditional moments of multi-horizon returns under a regime-switching model. These moments are applied to examine the relevance of risk horizon and regimes for buy-and-hold investors. We analyze the impact of time-varying expected returns and risk (variance and covariance) on portfolio allocations' "term structure"—portfolio allocations as a function of the investment horizon. Using monthly observations on S&P composite index and 10-year Government Bond, we find that the term structure of the optimal allocations depends on market conditions measured by the probability of being in bull state. At short horizons and when this probability is low, buy-and-hold investors decrease their holdings of risky assets. We also find that the conditional optimal portfolio performs quite well at short and intermediate horizons and less at long horizons.



## 1. Introduction

Since the seminal work of Hamilton (1989, *Econometrica*), Markov switching models have been increasingly used in financial time-series econometrics because of their ability to capture certain key features, such as heavy tails, volatility clustering, and mean reversion in asset returns; see Cecchetti et al. (1990), Pagan and Schwert (1990), Turner et al. (1989), Gray (1996), Hamilton (1988), Timmermann (2000), Ang and Bekaert (2002), Guidolin and Timmerman (2008, 2005, 2006, 2007, 2008a), Dueker and Neely (2007), Taamouti (2009), Hamilton and Owyang (2010), Amisano and Geweke (2011) among others. In this paper, we use a Fourier transform to derive conditional and unconditional moments of a multivariate Markov regime-switching model. Further, we apply these popular models to study the relevance of conditional information for the asset allocations' term structure.

Our first and main contribution is we derive multivariate conditional and unconditional moments of multi-horizon returns using a Markov regime-switching model. This result can be viewed as a generalization to the multivariate case of certain moments derived in Timmermann (2000). The latter derives the unconditional moments of univariate simple and

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autoregressive Markov switching models. He characterizes the patterns of volatility, skewness and kurtosis produced by these models as a function of the transition probabilities and parameters of the underlying state densities entering the switching process. Finally, he derives the autocovariance of the level and squares of time series generated by the Markov switching processes.

There are at least four main differences between the present paper and Timmermann's (2000) paper. First, here we consider a multivariate rather than a univariate Markov switching model, but we only focus on a simple Markov switching model without allowing for autoregressive lags as in Timmermann (2000). Multivariate models allow us to capture the dependence between the univariate processes. Second, we derive the moments of both simple and aggregated returns, whereas Timmermann (2000) only focuses on simple returns. Third, we derive both conditional and unconditional moments, whereas Timmermann (2000) only derives the unconditional moments. We use Hamilton filter to account for conditional information. Finally, we use a Fourier transform which is a natural way to derive conditional and unconditional moments.

Our second contribution is we apply the multivariate conditional and unconditional moments derived before to examine the implications of risk horizon and regimes for buy-and-hold investors. We also evaluate the importance of conditional information for the optimal portfolio performance. In a recent study, Campbell and Viceira (2005) [see also Campbell (1991), Hodrick (1992), Campbell and Viceira (1999), Barberis (2000), and Campbell et al. (2003)] use a linear vector autoregressive model (VAR) to investigate the implications for risk across investment horizons of time variation in the expected returns. They derive the conditional moments of multi-horizon returns that are generated by linear VAR model and they show how to extract the term structure of risk corresponding to a mean–variance portfolio. Empirically, they find that changes in investment opportunities can alter the risk-return trade-off of bonds, stocks, and cash across investment horizons.

However, Campbell and Viceira (2005) ignore the time variation in volatility and focus on the time variation in the expected returns, which is due to the linear autoregressive (AR) dynamics. Further, their analysis is based on the examination of the unconditional average portfolio allocations rather than the full range of allocations, which would be optimal under different market conditions. In contrast, in the present paper, we first allow for time variation in both mean and variance–covariance matrix of vector of returns. The time variations in these two moments are due to the regime switching with time-varying transition probabilities. As we know, regime-switching is able to capture certain aspects of the economic business cycle that reflects movements in aggregate macroeconomic variables. Thus, Markov regime-switching may contain information about aggregate macro variables. Second, we analyze the full range of allocations under different market conditions that are measured by the conditional (filtered) probability of regimes. Finally, we examine the relevance of both risk horizon and regimes for buy-and-hold investors.

In another related paper, Guidolin and Timmermann (2007) use regime-switching to study the asset allocation decisions. The fundamental difference between the present paper and Guidolin and Timmermann's paper is here we focus on conditional portfolios, based on conditional (filtered) probability of regimes, whereas Guidolin and Timmermann (2007) consider conditional on regimes and unconditional portfolios. In other words, we look at the term structure of the optimal portfolio given a probability of regime, whereas Guidolin and Timmermann look at the term structure of the optimal portfolio assuming that the investors observe the regimes or that they are at steady-state. In practice, the difference between our and their analysis is crucial, since as it is shown in the present paper the conditional portfolio performs much better than the unconditional one, especially at short and intermediate horizons.

To concentrate on the implications of risk horizon and regimes for buy-and-hold investors, we abstract from several other considerations that may be important in practice. Our analysis is based on a mean–variance framework, thus we ignore the risk model related to the departure from normality assumption and the possibility that investors care also about other return distribution properties.

Using monthly observations on S&P composite index and 10-year Government Bond, we first find that the term structure of the optimal allocations depends on the market conditions measured by the probability of being in bull state. At short horizons and when this probability is low, buy-and-hold investors decrease their holdings of risky assets. Second, we find that the conditional optimal allocations to stock and bond may be increasing or decreasing at short and intermediate horizons and converge to the unconditional optimal allocations at long horizons. Finally, we find that the conditional optimal portfolio performs quite well at short and intermediate horizons and less at long horizons.

The remainder of this paper is organized as follows. In Section 2 we introduce notations and we define our model. In Section 3 we derive the conditional characteristic function and the conditional moments of vector of multi-horizon returns. A description of the data and the empirical results are given in Section 4. We conclude in Section 5. Other results and technical proofs are presented in the Appendices A1–A3.

## 2. Framework

We assume there are $n$ risky assets plus one riskless asset in the economy. We denote by $r_t=(r_{1t},\ r_{2t},\dots,\ r_{nt})^\top$ the vector of risky asset returns. We consider the following notations:

$$\zeta_t = \begin{cases} (1,0,0,\dots,0)^\top \text{ when } s_t = 1 \\ (0,1,0,\dots,0)^\top \text{ when } s_t = 2 \\ (0,0,0,\dots,1)^\top \text{ when } s_t = N, \end{cases} \tag{1}$$