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Min–max constrained quasi-infinite horizon model predictive control using linear programming

D. Megías[a,*], J. Serrano[a], C. de Prada[b]

[a]Unitat d'Enginyeria de Sistemes i d'Automàtica, Departamento d'Informàtica, ETSE, Universitat Autònoma de Barcelona, Edifici Q, 08193 Bellaterra (Cerdanyola del Vallès), Barcelona, Spain

[b]Departamento de Ingeniería de Sistemas y Automática, Universidad de Valladolid, 47011 Valladolid, Spain

Abstract

In this paper a quasi-infinite horizon 1-norm GPC is presented. This controller, combined with a global uncertainty description and an uncertainty band-updating procedure, has led to a robust algorithm with extremely low computational requirements. Only a linear programming (LP) problem needs to be solved to compute a control profile. This scheme can be successfully applied even to hard non-linear systems with relatively fast dynamics, as the large computational burden related to non-linear model predictive controllers is avoided. Simulation results performed on several constrained non-linear systems are provided.

Keywords: Predictive control; Uncertainty; Robustness; Min–max techniques

1. Introduction

Model, model-based or receding-horizon predictive control (MPC or RHPC) has become a mature control strategy in the last few years. One of the main reasons for MPC success is the possibility to include constraints in a systematic manner within the control design. All physical systems are constrained in some way (plant or actuator saturations, security limits, etc.) and this situation must be taken into account by the controller. After the first few succeeding works, the lack of stability guarantees [1] and robustness results in MPC was highlighted by several authors.

This paper is focused on transfer function formulations which are closely related to the generalised predictive control (GPC) of [2].

Stabilising algorithms, such as the constrained receding-horizon predictive controller (CRHPC) [3,4], the stable GPC (SGPC) [5] and the infinite horizon approach (e.g. the infinite horizon GPC or GPC^{∞}) [6,7] are now available. The CRHPC is based on constraining[1] the predicted output of the model to match the setpoint for a number of samples. The SGPC provides the same closed-loop transfer functions as the CRHPC, but enjoys better numerical properties. Finally, infinite horizon controllers are based on minimising a cost function over an infinite prediction horizon.

Stabilising approaches exist also in the non-linear model MPC family, as reported in [8]. These schemes involve a large computational burden which makes them impractical in many real control situations, especially if fast dynamics occur. The time constants of many industrial processes (as some of the examples provided below illustrate) are of seconds or a few minutes. In such a case, it is not often possible to apply non-linear MPC methods due to the amount of on-line computations they require. However, many non-linearities may be modelled (at least about some operating point) as a disturbance affecting a linear system. This approach makes it possible to formulate a min–max optimisation problem which can be solved on-line with a less computationally intensive method compared to non-linear MPC. This alternative provides a somewhat more conservative solution, but it allows to satisfy constraint specifications as well as non-linear MPC does and often achieves a similar performance. In this paper, the min–max problem is posed as a linear programming (LP) problem, which can be solved very efficiently with standard tools. In some examples, the solution

* Corresponding author. Tel.: +34-935-81-3027; fax: +34-935-81-3033.

E-mail addresses: david.megias@uab.es (D. Megías), javier.serrano@uab.es (J. Serrano), prada@autom.uva.es (C. de Prada).

[1] Henceforth, the term "constrained" is only used for inequality input/output/state constraints. Equality constrained methods with no inequality constraints are referred to as "unconstrained".

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is obtained in just a fraction of a second. This is the reason for focusing this paper on linear model formulations.

It is not enough that the controller stabilise the nominal system, because model/plant mismatch always occurs. The first few robustness results in the MPC framework [5,9–11] were obtained for unconstrained, linear model predictive controllers in the single-input/single-output (SISO) case, and do not consider the possibility of non-linear or time-varying uncertainty. Newer results [12] make use of polytopic linear model descriptions (or structured plant uncertainty) and linear matrix inequalities (LMI) to design efficient robust min–max controllers which satisfy input, output and state constraints, with stability guarantees. This method can even be applied to non-linear systems, but with a few limitations. Firstly, if a constant setpoint tracking problem is considered, the results hold only for uncertain linear time-invariant (LTI) systems. Secondly, strong non-linearities (such as saturation, hysteresis, relay or deadzone) can occur in a plant. In this case a polytope of linear models is not a convenient representation. And finally, unmeasurable disturbances may appear in such a way that they are not included within this kind of description, leading to inaccurate predictions and, possibly, to constraint violation. In addition, these techniques are difficult to extend to transfer function formulations.

When any of the situations reported above is relevant, a global uncertainty description can be an alternative. A global uncertainty is an unknown (bounded) quantity which, added to the model output/state, produces the true system output/state. This very simple concept is general enough to range over linear and non-linear, time-varying and time invariant, stable and unstable uncertainty. In addition, it perfectly describes (unmeasurable) disturbances. Min–max algorithms, either for state-space [13] or transfer function [14,15] models, can be easily developed using this formulation, and it is even possible to write them as an efficiently solvable LP problem.

The scope of this paper is to present and test predictive control schemes based on a global uncertainty description, and to compare them with some classical robustness enhancing tools. The paper is organised as follows. Section 2 formulates a 1-norm quasi-infinite horizon GPC controller ($QGPC_1^\infty$). In Section 3, the min–max problem based on the global uncertainty approach is formulated, and the min–max $QGPC_1^\infty$ is obtained. Section 4 presents a set of simulated experiments performed on several non-linear plants, including hard non-linearities and two different chemical reactors. Finally, Section 5 finishes the paper summarising the most significant concluding remarks. The formulae provided in the sequel are written for the SISO case only for simplicity of notation, but all of them can be readily extended to the multiple-input/multiple-output (MIMO) case in a straightforward manner.

2. Quasi-infinite horizon 1-norm GPC

For min–max controllers, a 1-norm cost function leads to an optimisation problem which is much less computationally expensive than a 2-norm counterpart, as remarked in [14]. By using the 1-norm, the min–max optimisation becomes just an LP problem with a number of constraints which grows only linearly with the prediction horizon. That is the reason for which a quasi-infinite horizon 1-norm GPC — referred to as $QGPC_1^\infty$ hereafter — is presented in this section. The CRHPC could be an alternative to GPC^∞, but it tends to produce a suboptimal deadbeat-like behaviour, especially when short horizons are used, and leads to lower robustness margins compared to the infinite horizon approach [11]. In addition, the infinite prediction horizon of GPC^∞ becomes a short finite one after a few manipulations [7], which is very convenient for the min–max procedure depicted in Section 3.

Let the process model be given by

$$A\left(q^{-1}\right)y(t) = B\left(q^{-1}\right)u(t-1) + \frac{T\left(q^{-1}\right)}{\Delta}\xi(t),$$

where $u(t)$ and $y(t)$ are, respectively, the output and the input signals, $A(q^{-1})$, $q^{-1}B(q^{-1})$ and $T(q^{-1})$ are polynomials[2] in the backward shift operator q^{-1}, with degrees n_a, n_b and n_t, respectively, $\xi(t)$ is a zero-mean, stochastic disturbance signal, and $\Delta = 1 - q^{-1}$ is the discrete differencing operator. In addition, let $A(q^{-1}) = \overline{A}(q^{-1})\tilde{A}(q^{-1})$ be a decomposition of strictly stable ($\overline{A}$) and unstable ($\tilde{A}$) factors of the denominator, with $\overline{A}(q^{-1}) = 1 + \overline{a}_1 q^{-1} + \cdots + \bar{a}_{n_{\overline{a}}} q^{-n_{\overline{a}}}$, and $\deg(\tilde{\boldsymbol{A}}) = n_{\tilde{a}}$.

An optimal control move sequence can be computed to minimise a (1-norm) cost function such as

$$J^\infty(t) = \sum_{j=1}^{\infty} \mid e(t+j \mid t) \mid + \rho \sum_{j=1}^{N_u} \mid \Delta u(t+j-1 \mid t) \mid,$$

where N_u is the control horizon ($\Delta u(t+j \mid t)$ is 0 for $j \geqslant N_u$), $e(t+j \mid t) \doteq w(t+j \mid t) - y(t+j \mid t)$ are the predicted tracking errors, $w(t+j \mid t)$ are the future values of the setpoint, $y(t+j \mid t)$ are predictions of the output, and ρ is a positive weighting (which has been considered constant only for simplicity of notation, but could be a j-function). Only the first control move is used to update the control signal, which is the usual receding-horizon strategy. The controller resulting of the minimisation of $J^\infty(t)$ is stabilising, since the stability proof provided by Scokaert [7] holds despite the use of the "modulus" instead of the "square" in the definition of $J^\infty(t)$ (a stability proof is provided in [15]). However, an infinite dimensional

[2] In the min–max formulation of Section 3, the disturbance model is replaced by a different uncertainty description which does not involve the observer polynomial T.