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Duality in fuzzy multi-criteria and multi-constraint level linear programming: a parametric approach

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Abstract

This paper presents a parametric approach for duality in fuzzy multi-criteria and multi-constraint level linear programming (MC^2LP) which extends fuzzy linear programming approaches. First, the MC^2-simplex method is used to solve the crisp prima–dual MC^2LP pair and then, through these crisp formulations, separate membership functions are constructed for fuzzy primal and dual program by considering the corresponding primal and dual decisions. For each program, a set of fuzzy potential solutions is determined in terms of the membership function and the related optimal solution with a certain range of decision parameters. Finally, using the primal and dual membership functions, a fuzzy weak duality function is obtained for any pair of primal and dual fuzzy potential solutions.

Keywords: Multi-criteria linear programming; Multi-criteria and multi-constraint level linear programming; Fuzzy linear programming; Parametric linear programming; Duality

1. Introduction

The multiple criteria and multiple constraint level linear programming (MC^2LP) is an extension of linear programming (LP) and multiple criteria linear programming (MCLP). Since the traditional linear programming problem is formulated with only a single criterion (objective) and a single constraint (resource availability) level (right hand side), it has limitations in dealing with real world problems that involve multiple conflicting criteria. The MC linear programming can be used to handle some problems with multiple conflicting criteria. However, it cannot be used to deal with problems with both multiple conflicting criteria and multiple constraint levels. For example, the problem of selecting optimal linear production system designs is a decision problem with multiple conflicting criteria and multiple constraint levels. Multiple conflicting criteria are maximizing the total sales of selected products, total profit, and customer satisfaction, while multiple constraint levels are determined by a group of decision makers, such as the vice president for finance, the vice president for production, and the vice president for marketing. Either linear programming or MC linear programming cannot solve such a problem. But, it can be formulated as an MC^2LP problem. The MC^2LP was originally

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defined and solved by Seiford and Yu [11]. The solution method has been completely discussed in Yu [16] and Shi [12]. For a given MC^{2}LP problem, we can identify a set of potential solutions .by using the MC^2-simplex method of [11]. Each potential solution optimizes the MC^2 problem under a certain range of decision parameters that are the criteria and constraint level weight vectors. Given any values of decision parameters, there is also a potential solution optimizing the MC^2 problem. Thus, the set of potential solutions can potentially optimize the MC^2 problem for all possible changes of decision parameters.

In reality, due to the complicated nature of some decision problems with the MC^2 framework, it may be difficult to find an optimal solution that satisfies the decision makers. To overcome this difficulty, Shi and Liu [14] and Liu and Shi [8] have studied fuzzy MC^2 problems. Instead of finding a set of potential solutions for an MC^2 problem, they consider the decision makers' goal-seeking and compromise behavior to approach a set of "satisficing solutions" between an upper and a lower aspiration level. These aspiration levels can be represented by the upper and lower bounds of acceptability for objective payoffs. In addition, Liu et al. [9] have explored a constructive approach to a duality of fuzzy MC^{2}LP problems. In these approaches, both fuzzy primal and dual programs of a given MC^2 problem are defined as the problems with a fuzzy optimal objective, in which the decision makers have a goal-seeking and compromise behavior for a satisficing solution [9,12,14,16]. Thus, the fuzzy optimal objective problem is fuzzy-objective MC^2 problem, whose satisficing solution is a fuzzy potential solution with a satisficing level between an upper aspiration bound and a lower aspiration bound of the objective payoff. The MC^2-simplex method is first used to locate sets of potential solutions of maximum and minimum MC^2 problems for primal and dual programs, respectively. Then the separate membership functions for primal and dual programs are constructed. For each program, a set of fuzzy potential solutions is determined in terms of the membership functions and "primal" potential bases with certain ranges of decision parameters.

In this paper, we alternatively explore to study duality of fuzzy MC^{2}LP problem with fuzzy objective and constrain by a parametric approach. It differs from the known constructive approach [9] and is an extension of the approach of the duality of fuzzy LP [2,10,18]. For a given MC^2 problem, its fuzzy primal and dual programs are defined as the problems with fuzzy objective and constrain. We first use the MC^2-simplex method to locate sets of potential solutions with respect to their crisp problems, respectively. Then we construct separately membership functions for primal and dual programs. For each program, a set of fuzzy potential solutions is determined in terms of the membership functions and maximizing solutions (decisions) with certain ranges of decision parameters. Finally, we construct a fuzzy weak duality function for any pair of primal and dual fuzzy potential solutions.

This paper proceeds as follows. In Section 2, to facilitate the discussion for the duality, we briefly sketch the framework of MC^{2}LP. In Section 3, we review duality of fuzzy LP. In Section 4, we present a parametric approach to duality of fuzzy MC^{2}LP. In Section 5, we conclude this paper with some further research problems.

2. MC^2 linear programming

In this section, we sketch the basic concepts of MC^{2}LP (see [11,12,16]). An MC^{2}LP problem can be formulated as

$$\begin{aligned} \text{Max}\quad & \lambda^t \mathbf{C}\mathbf{x} \\ \text{s.t.}\quad & \mathbf{A}\mathbf{x} \leqslant \mathbf{D}\gamma, \\ & \mathbf{x} \geqslant \mathbf{0}, \end{aligned} \tag{1}$$

where $\mathbf{C}\in R^{q\times n}, \mathbf{A}\in R^{m\times n}$ and $\mathbf{D}\in R^{m\times p}$ are matrices of dimensions $q\times n$, $m\times n$, and $m\times p$, respectively; $\mathbf{x}\in R^n$ is decision variable, $\lambda>\mathbf{0}$ is called the q-dimensional criteria weight vector and $\gamma>\mathbf{0}$ is the p-dimensional constraint level weight vector. Both (γ,λ) are assumed to be unknown.

Given basic variables $\{x_{j_1},\ldots,x_{j_m}\}$ for problem (1), we denote the index set of the basic variables by $J=\{j_1,\ldots,j_m\}$. Then, we can write $\mathbf{x}(J)=[x_{j_1}\cdots x_{j_m}]^t$. Note that $\mathbf{x}(J)$ may contain slack variables. Without confusion, J is called a basis for problem (1). Given a basis J with its basic variables $\mathbf{x}(J)$, we define the associated basis matrix $\mathbf{B}_J$ as the sub-matrix of $[\mathbf{A},\mathbf{I}]$ in (1) with column index of J (i.e. column j of $[\mathbf{A},\mathbf{I}]$ is in $\mathbf{B}_J$ if and only if $j\in J$), and