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A weight space-based approach to fuzzy multiple-objective linear programming

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Abstract

In this paper, the effects of uncertainty on multiple-objective linear programming models are studied using the concepts of fuzzy set theory. The proposed interactive decision support system is based on the interactive exploration of the weight space. The comparative analysis of indifference regions on the various weight spaces (which vary according to intervals of values of the satisfaction degree of objective functions and constraints) enables to study the stability and evolution of the basis that correspond to the calculated efficient solutions with changes of some model parameters.

Keywords: multiple-objective linear programming; Fuzzy sets; Interactive methods; Weight space; Efficient solutions; Decision support systems

1. Introduction

Most of realistic decision-making problems, essentially those stemming from complex and ill-structured situations, are characterized by the existence of multiple, conflicting and incommensurate objectives and are subject to the unavoidable influence of distinct sources of uncertainty. Therefore, models must take into account vague information, imprecise requirements, modifications of the original input data, imprecision stemming from the modeling phase, needed simplifications, unexpected occurrence of important events and the subjective and evolutive nature of human preference structures whenever multiple objectives and trade-offs are at stake.

Interactive techniques based on the weight space decomposition have been developed and computationally implemented as the core of a decision support system (DSS) to deal with uncertainty in multiple-objective linear programming (MOLP) models by using fuzzy set theory concepts.

The decision maker (DM) has the possibility of interactively changing the membership functions associated with the mathematical constraint relations and the objective functions optimization. It is then possible to evaluate the effects of changing the model parameters and to study alternative scenarios without having to reformulate the problem.

The comparative analysis of the weight spaces corresponding to distinct satisfaction degrees is a

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valuable tool to study the fuzzy efficient solution set. Among these fuzzy solutions, the DM may choose a satisfactory compromise one according to his/her preference structure which may change as more knowledge about the problem is acquired throughout the interactive decision aid process.

This paper is organized in five sections. The introduction of the main concepts of fuzzy multiple-objective linear optimization problems is made in Section 2. The conceptual aspects of the proposed DSS are presented in Section 3. The example presented in Section 4 aims at illustrating the concepts presented. Some conclusions about the potentialities of this approach are drawn in Section 5.

2. Decision making in a fuzzy environment

In classical mathematical programming, multiple-objective problems are concerned with the optimization of multiple, conflicting and incommensurate objective functions subject to constraints representing the availability of limited resources and/or requirements.

The following MOLP problem is considered in this study:

$$\max\ \underline{\mathbf{f}}(\underline{\mathbf{x}}) = C\underline{\mathbf{x}} \tag{1}$$

s.t.

$$\left.\begin{array}{l} A\,\underline{\mathbf{x}}\{\leq\ =\ \geq\}\underline{\mathbf{b}} \\ \quad\underline{\mathbf{x}}\geq\underline{\mathbf{0}} \end{array}\right\} X$$

where $\underline{\mathbf{x}}\in\mathbb{R}^n$ is the decision variable vector, $\mathbf{C}\in\mathbb{R}^{p\times n}$ is the objective function matrix, $A\in\mathbb{R}^{m\times n}$ is the technological matrix and $\underline{\mathbf{b}}\in\mathbb{R}^m$ is the right-hand side vector.

Constraints separate all possible solutions into two distinct sets: those which are feasible (X) and those which are not feasible. Objective functions are to be pursued to the greatest possible extent with regard to the feasible region. However, since the objective functions are generally in conflict, there is not usually a solution that optimize all the objective functions at the same time. The concept of optimal solution to a single objective problem gives, thus, place in a multiple-objective context to the concept of efficient solutions: feasible solutions for which no improvement in any objective function is possible without sacrificing on at least one of the other objective functions. These problems entail analyzing trade-offs among the objectives in order to get a satisfactory compromise from the set of efficient solutions.

Let us consider p objective functions $\underline{\mathbf{f}}(\underline{\mathbf{x}})=(\boldsymbol{f}_1(\underline{\mathbf{x}}), \boldsymbol{f}_2(\underline{\mathbf{x}}), \ldots, \boldsymbol{f}_p(\underline{\mathbf{x}}))$, which are to be maximized in a feasible region $\boldsymbol{X}$.

$\underline{\bar{\mathbf{x}}}\in\boldsymbol{X}$ is an efficient solution, if and only if no $\underline{\hat{\mathbf{x}}}\in\boldsymbol{X}$ exists such that

$$f_k(\underline{\hat{\mathbf{x}}})\geq f_k(\underline{\bar{\mathbf{x}}}),\ \text{for}\ k=1,\ \ldots,p\quad \text{and}$$
$$f_k(\underline{\hat{\mathbf{x}}}) > f_k(\underline{\bar{\mathbf{x}}}),\ \text{for at least one}\ k=1,\ \ldots,p \tag{2}$$

The concept of efficient solution generally refers to the variable space whereas the nondominance concept refers to the corresponding image in the objective function space. That is, if $\underline{\mathbf{x}}$ is efficient then $\underline{\boldsymbol{f}}(\underline{\boldsymbol{x}})$ is nondominated.

In a fuzzy environment, the main purpose is to find the "most satisfactory" solution under incomplete, subjective, imprecise and/or vague information. In the symmetric model proposed by Bellman and Zadeh [1], there is no difference between objectives and constraints. A fuzzy decision can be viewed as a fuzzy set $\tilde{D}$ resulting from the intersection of fuzzy goals $\tilde{G}_k$ and fuzzy problem constraints $\tilde{C}_i$

$$\tilde{D} = \tilde{G}_1 \cap \tilde{G}_2 \cap \ldots \cap \tilde{G}_p \cap \tilde{C}_1 \cap \tilde{C}_2 \cap \ldots \cap \tilde{C}_m \tag{3}$$

An optimal decision is an element with maximum degree of membership to this set. Generally, the most convenient way to model intersection is the minimum operator. If all membership functions $\mu_j(\underline{\mathbf{x}})$ are known in a space of alternatives X ($\mu_j(\underline{\mathbf{x}})$: $X\rightarrow[0,1]$), then the fuzzy decision is defined by:

$$\begin{aligned}\mu_{\tilde{D}}(\underline{\mathbf{x}}) &= \min\{\mu_{\tilde{G}_1}(\underline{\mathbf{x}}), \mu_{\tilde{G}_2}(\underline{\mathbf{x}}), \ \ldots, \mu_{\tilde{G}_p}(\underline{\mathbf{x}}), \\ &\quad \mu_{\tilde{C}_1}(\underline{\mathbf{x}}), \mu_{\tilde{C}_2}(\underline{\mathbf{x}}), \ \ldots, \mu_{\tilde{C}_m}(\underline{\mathbf{x}})\} \\ &= \min\{\mu_j(\underline{\mathbf{x}})\},\ \text{for all}\ \underline{\mathbf{x}}\end{aligned} \tag{4}$$

and the optimal decision by:

$$\max\ \mu_{\tilde{D}}(\underline{\mathbf{x}}) = \max[\min\{\mu_j(\underline{\mathbf{x}})\}],\ \text{for all}\ \underline{\mathbf{x}} \tag{5}$$

Werners [8,9] proposed the generalization of the classical efficient solution definition for the fuzzy multiple-objective linear programming (FMOLP)