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Satisficing solutions and duality in interval and fuzzy linear programming

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Abstract

In this paper, we introduce a class of fuzzy linear programming problems and define the concepts of feasible and satisficing solutions—the necessary tools for dealing with such problems. In this way, we show that the class of crisp (classical) LP problems can be embedded into the class of FLP ones. Moreover, for FLP problems we define the concept of duality and prove the weak and strong duality theorems. Further, we define a class of interval linear programming problems as a special subclass of FLP problems and apply the previous results to this special case. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

This work has been motivated by the former papers [6–8], where modality indices are applied to fuzzy preference relations in fuzzy linear programming.

An interest of operations researchers focused on linear programming problems (LP problems) with uncertainty in coefficients was launched probably by Danzig's book [1] published in 1963. Up till now, there exist a lot of papers dealing with fuzzy linear programming problems (FLP problems), see, e.g. the survey paper [23] or [13]. The authors of this paper also contributed to this area, see e.g. [6–9] or [16–20]. Various approaches to FLP problems, that can be found in the literature, could be characterized by a common feature: by the use of fuzzy sets, a FLP problem is formulated,

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then it is transformed to a special LP problem which is further studied and solved by the tools and technology of LP.

In this paper, we employ a reverse approach. We first introduce a broad class of FLP problems and define the concepts of feasible and satisficing solutions—the necessary tools for dealing with such problems. In this way, we show that the class of classical LP problems can be embedded into the class of FLP ones. Moreover, for FLP problems we define the concept of duality and prove the weak and strong duality theorems. Further, we define a class of interval linear programming problems (ILP problems), a special subclass of FLP problems and apply the previous results to this special case.

In the first part (Section 2) of this paper we give some preliminary results for fuzzy relations which will be used in the second part (Section 3) for FLP problems. The proofs of propositions in Section 2 are omitted, the reader can find them in [19,21] or in a different setting in [6].

2. Fuzzy relations and their properties

2.1. Notation—Binary and valued relations

Let X be a nonempty set. By $\mathscr{F}(X)$ we denote the set of all fuzzy subsets A of X, where every fuzzy subset A of X is uniquely given by the membership function $\mu_A : X \to [0,1]$. If μ_A is a characteristic function, i.e. $\mu_A : X \to \{0,1\}$, we say that the fuzzy subset A is *crisp*. Instead of μ_A, the characteristic function is denoted by χ_A. All crisp subsets of X are identified with subsets of X.

Let A be a fuzzy subset of X. The *height of* A, $Hgt(A)$, is given by

$$Hgt(A) = \sup\{\mu_A(x) \mid x \in X\}.$$

We say that a fuzzy subset A of X is *normal* if the *core of* A,

$$Core(A) = \{x \in X \mid \mu_A(x) = 1\},$$

is nonempty, i.e. there exists $\bar{x} \in X$ with $\mu_A(\bar{x}) = 1$.

A fuzzy subset ${}^cA$ of X with the membership function $\mu_{{}^cA}$ defined for all $x \in X$ by

$$\mu_{{}^cA}(x) = 1 - \mu_A(x) \tag{1}$$

is called the *complement of* A.

For a fuzzy subset A of X with the membership function $\mu_A : X \to [0,1]$ and $\alpha \in [0,1]$, we define the α-*cut* $[A]_\alpha$ (α-*level set*) and *strict* α-*cut* $(A)_\alpha$ (*strict* α-*level set*) as

$$[A]_\alpha = \{x \in X \mid \mu_A(x) \geqslant \alpha\}$$

and

$$(A)_\alpha = \{x \in X \mid \mu_A(x) > \alpha\},$$

respectively.

Let X be a normed linear space. A fuzzy subset A of X is *closed*, *bounded*, *compact* or *convex*, if the α-*cuts* $[A]_\alpha$ are closed, bounded, compact or convex subsets of X for every $\alpha \in (0,1]$, respectively.

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