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Fuzzy Sets and Systems 135 (2003) 123–150

FUZZY sets and systems

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Oblique fuzzy vectors and their use in possibilistic linear programming ☆

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Abstract

In this paper, we propose oblique fuzzy vectors to treat the interactivity among fuzzy numbers. Oblique fuzzy vectors are extensions of fuzzy numbers and vectors of non-interactive fuzzy numbers. The interactivity among fuzzy numbers can be treated by a non-singular matrix in an oblique fuzzy vector. We discuss characterization of an oblique fuzzy vector and the tractability of manipulation of oblique fuzzy vectors in fuzzy linear functions. Moreover, we discuss possibilistic linear programming problems with oblique fuzzy vectors. It is shown that the possibilistic linear programming problems are reduced to linear programming problems with a special structure.

Keywords: Interactive fuzzy numbers; Linear programming; Extension principle; Necessity measure; Benders decomposition method

1. Introduction

Interval arithmetic [10] and calculations of fuzzy numbers [3] have been developed mainly under the assumption of non-interactivity among uncertain variables. In interval analysis, intervals are used for two purposes, i.e., (1) to enhance the accuracy of computer calculation (a countermeasure of round-off errors by floating point method) and (2) to treat uncertain parameters whose ranges are known as intervals. In fuzzy systems theory, fuzzy numbers are used mainly for the second purpose of interval analysis, i.e., to treat uncertain parameters whose ranges are known as fuzzy numbers.

☆ This research is partly supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid Scientific Research for Encouragement of Young Scientists, No. 12780333.

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0165-0114/03/$ - see front matter
PII: S0165-0114(02)00252-X

For the first purpose in interval analysis, there is little importance in introduction of interactivity among uncertain variables. On the other hand, for the second purpose, the non-interactivity assumption is too restrictive to represent real situations. Indeed, some unsuitable results are obtained in possibilistic portfolio selection problems because of the non-interactivity assumption (see, for example, [7]). Therefore, it is required to introduce the interactivity among uncertain variables to interval arithmetic and calculations of fuzzy numbers.

However, in general, the calculation with interactive fuzzy numbers is much more difficult and complex than that with non-interactive fuzzy numbers. In applications of interactive fuzzy numbers to possibilistic linear programming, to fuzzy linear regression and, etc., the tractability of linear calculation is advantageous. Indeed, an advantage of possibilistic linear programming over stochastic linear programming is the linearity with respect to decision variables in calculations of linear functions with uncertain coefficients (see, for example, [6]). Therefore, for these methodologies, it is better to preserve the tractability, i.e., linearity in linear calculation of fuzzy numbers.

Dubois and Prade [2] and Tanaka and Ishibuchi [13] showed that, in some special cases of interactive fuzzy numbers, the sum and linear function values of interactive fuzzy numbers can be obtained easily. The case treated by Dubois and Prade [2] preserves the linearity to a certain extent in calculation of sum of fuzzy numbers but it is rather restrictive to be applied to the representation of interactive fuzzy numbers in real problems. On the other hand, interactive fuzzy numbers treated by Tanaka and Ishibuchi [13] have a quadratic membership function and the linear function values of those fuzzy numbers are not obtained by linear calculation but by quadratic calculation.

In this paper, we propose a model of interactive fuzzy numbers that preserves the linearity to a certain extent. In order to treat the interactivity, we use a non-singular matrix called an obliquity matrix and we assume the uncertain variables are non-interactive under the linear transformation by the obliquity matrix. Similar approach was proposed by Ramík and Nakamura [12] and Miyakawa et al. [9] by the name of canonical fuzzy numbers. The model proposed in this paper is an extension of the canonical fuzzy numbers and named oblique fuzzy vectors. Not only canonical fuzzy numbers but also vectors of non-interactive fuzzy numbers, fuzzy numbers and intervals are covered by the concept of oblique fuzzy vectors as special cases.

We also discuss some characterizations of oblique fuzzy vectors. Some conditions for oblique fuzzy vectors to be characterized by marginal fuzzy numbers and a non-singular matrix are shown. Calculation of linear function values of oblique fuzzy vectors is investigated. We show that linear function values of oblique fuzzy vectors can be calculated easily without a great loss of linearity.

Moreover, possibilistic linear programming problems with oblique fuzzy vectors are discussed. The problems are formulated by using necessity measures. Utilizing the result of a fuzzy linear function with an oblique fuzzy vector, it is shown that the formulated problems can be reduced to linear programming problems. From a special structure of the obtained linear programming problems, an algorithm based on Benders decomposition is presented.

This paper is organized as follows. In Section 2, we review briefly the calculations of non-interactive fuzzy numbers and properties of necessity measures as preliminaries. Oblique fuzzy vectors are defined and their properties are investigated in Section 3. In Section 4, we discuss a possibilistic linear programming problem with oblique fuzzy vectors. Finally, some concluding remarks are given.