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A decision support methodology for stochastic multi-criteria linear programming using spreadsheets

David C. Novak [a,1], Cliff T. Ragsdale [b,*]

[a]Department of Operations and Information Management, University of Connecticut, 1200 Hillside Road, Unit 1041 OPIM, Storrs, CT 06269-1041, USA

[b]Department of Business Information Technology, Virginia Tech, 1007 Pamplin Hall, Blacksburg, VA 24061, USA

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Abstract

In recent years, tools for solving optimization problems have become widely available through the integration of optimization software (or solvers) with all major spreadsheet packages. These solvers are highly effective on traditional linear programming (LP) problems with known, deterministic parameters. However, thoughtful analysts may rightly question the quality and robustness of optimal solutions to problems where point estimates are substituted for model parameters that are stochastic in nature. Additionally, while many LP problems implicitly involve multiple objectives, current spreadsheet solvers provide no convenient facility for dealing with more than one objective. This paper introduces a decision support methodology for identifying robust solutions to LP problems involving stochastic parameters and multiple criteria using spreadsheets.

Keywords: Stochastic programming; Multi-criteria optimization; Decision support; Spreadsheets

1. Introduction

Linear programming (LP) is a mathematical programming technique designed to optimize a (single) linear objective function subject to a linear constraint set where all model parameters are assumed to be known with certainty. Over the past several decades, numerous applications for LP have been proposed for improving the efficiency of business operations [26]. Unfortunately, uncertainty in such things as costs, demand, interest rates, production yields, and equipment reliability is the practical reality faced by business decision makers (DMs) on a daily basis. As a result, some degree of randomness or uncertainty is likely to impact the parameter estimates used in medium- to long-term LP planning models and may also affect short-term models [6]. When one or more parameters in an LP problem are represented by a random variable, a stochastic LP problem results [26].

Effectively modeling the uncertainties in a stochastic LP model is a challenging yet necessary step in evaluating the robustness of a potential "optimal" solution to simultaneous changes in estimated parameter values. This is complicated by the fact that today's information-rich business environment impo-

* Corresponding author. Tel.: +1-540-231-4697.

E-mail addresses: dnovak@sba.uconn.edu (D.C. Novak), crags@vt.edu (C.T. Ragsdale).

1 Tel.: +1-860-486-1241.

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ses numerous competitive, political, financial, environmental, and societal pressures on DMs [4]. The inherent non-commensurate and conflicting nature of these pressures often makes it difficult or impossible to formulate a single objective function for many real-world decision problems. In these situations, multi-criteria optimization techniques must be used to allow DMs to properly model and analyze the trade-offs inherent with multiple conflicting objectives [29].

As managerial DMs become increasingly aware of the availability and purpose of the solver optimization software built into today's spreadsheets, questions about how to use this software with decision problems involving stochastic parameters and multiple criteria will likely emerge. A significant amount of research has been directed at developing techniques for solving stochastic programming problems. A similar level of effort has been devoted to challenging problems in multi-criteria optimization. Unfortunately, most of the resulting solution techniques are not easily understood by, available to, or implemented by practitioners in the business world. Additionally, relatively little work has focused on devising general solution procedures for optimization problems that are both stochastic and multi-criteria in nature.

The objective of this paper is to develop an easily understood methodology for solving stochastic, multi-criteria LP problems in spreadsheets. We begin with a review of the research literature followed by a description of our proposed methodology. We then illustrate this methodology using a stochastic, multi-criteria production planning problem implemented in Microsoft Excel. Finally, we highlight the benefits of our methodology and offer some closing comments.

2. Literature review

While the respective research literatures addressing stochastic programming and multi-criteria optimization are quite extensive, relatively little work has been done on developing solution methodologies for problems that are both stochastic and multi-criteria in nature. We briefly review the literature in each of these areas below and provide several references to point interested readers to appropriate sources of additional information.

2.1. Stochastic programming

In general, three approaches to stochastic programming have received considerable attention. The first approach yields problems with probabilistic or chance constraints that restrict the probability of infeasibility to be no greater than a prespecified threshold value [23]. Chance-constraint problems incorporate the uncertainty associated with specific events or conditions into the model by using probability constraints. Unfortunately, this approach can lead to problems with nonconvex constraint sets that can be extremely difficult to solve [26].

The second approach focuses on modeling future response or recourse. Recourse problems involve obtaining information about a process or system after the observation of some random event [14]. A response is modeled for each outcome (or group of outcomes) that is observed. Multi-stage recourse problems involve altering second-stage or reactive decisions based on the outcomes associated with first-stage decisions that are proactive in nature (before random outcomes are observed). A major difficulty in solving recourse problems is that the required calculation of the expected value of the recourse function involves multidimensional integration. Furthermore, for stochastic programming problems involving discrete random variables, this function is nondifferentiable [14,26].

Another well-known approach to solving stochastic programming problems involves scenario-based analysis [8]. The origin of these scenarios can be very diverse; they may be from a known discrete distribution, be obtained from limited sample information or some type of approximation scheme, or come from some type of preliminary analysis involving the probability of their occurrence based on the opinion of an expert [21]. While scenario-based approaches provide a relatively straightforward way to implicitly account for uncertainty, they may rely on an exceptionally large number of scenarios. Scenario-based approaches generally rely on either the a priori forecasting of all possible outcomes or the discretization of a continuous multivariate probability distribution resulting in an exponential number of scenarios. For example, the discretization of n uncertain variables with m discretion points results in m^n scenarios [25]. Thus, a problem with 20 uncertain variables and 3