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Duality in linear programming with fuzzy parameters and matrix games with fuzzy pay-offs

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Abstract

A dual for linear programming problems with fuzzy parameters is introduced and it is shown that a two person zero sum matrix game with fuzzy pay-offs is equivalent to a primal–dual pair of such fuzzy linear programming problems. Further certain difficulties with similar studies reported in the literature are discussed.

Keywords: Fuzzy numbers; Fuzzy matrix game; Fuzzy duality

1. Introduction

One of the most celebrated and useful result in the matrix game theory asserts that every two person zero sum matrix game is equivalent to two linear programming problems which are dual to each other. Thus, solving such a game amounts to solving any one of these two mutually dual linear programming problems and obtaining the solution of the other by using linear programming duality theory.

The earliest study of two person zero sum matrix game with fuzzy pay-offs is due to Campos [2] which still remains the most basic reference on this topic. Later Nishizaki and Sakawa [9] extended these ideas of Campos [2] to multiobjective matrix games as well. Though these studies have been motivated by the classical (crisp) two person zero sum matrix game theory but unlike their crisp counter parts, they do not take into consideration the fuzzy linear programming duality aspects and, therefore, do not seem to fully conceptualize the fuzzy matrix game model. In this context it may be noted that although certain fuzzy linear programming duality results are available

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(for example, [1,4,11]) such duality results for linear programming problems with fuzzy parameters have apparently not been reported in the literature. [1]

The basic aim of this paper is to first introduce duality in linear programming with fuzzy parameters and then have a relook of the fuzzy matrix game model considered by Campos [2] so as to analyze the same in the light of this duality. Specifically, it is shown that the procedure outlined by Campos [2] to solve such a game has certain inherent difficulties and it needs appropriate modifications and justifications for the various steps involved there in. The duality theory as introduced here plays a key role in the development of a modified procedure and its justification for solving such a game. In this context it may be emphasized that the purpose of this paper is not to generalize Campos' model [2] but rather to provide results which complement/supplement the basic ideas of [2].

The paper is organized as follows. Certain basic definitions and preliminaries with regard to crisp matrix games and fuzzy inequalities with fuzzy parameters are presented in Section 2. In Section 3, duality theory for linear programming problems with fuzzy parameters is introduced, while the main result, that a two person zero sum matrix game with fuzzy pay-offs is equivalent to an appropriate primal–dual pair of such fuzzy linear programming problems, is established in Section 4. Further, Section 5 discusses certain similarities and differences of the present study with that of Campos [2].

2. Definitions and preliminaries

Let R^n denote the n-dimensional Euclidean space and R^n_+ be its non-negative orthant. Let $A \in R^{m\times n}$ be an $(m \times n)$ real matrix and $\mathrm{e}^{\mathrm{T}} = (1, 1, \ldots, 1)$ be a vector of 'ones' whose dimension is specified as per the specific context.

By a (crisp) two person zero sum matrix game G we mean the triplet $G = (S^m, S^n, A)$ where $S^m = \{x \in R^m_+, \mathrm{e}^{\mathrm{T}}x = 1\}$ and $S^n = \{y \in R^n_+, \mathrm{e}^{\mathrm{T}}y = 1\}$. In the terminology of the matrix game theory, S^m(respectively, S^n) is called the *strategy space* for Player I (respectively, Player II) and A is called the *pay-off matrix*. Also it is a convention to assume that Player I is a maximizing player and Player II is a minimizing player. Further for $x \in S^m$, $y \in S^n$, the scalar $x^{\mathrm{T}}Ay$ is the pay-off to Player I and as the game G is zero sum, the pay-off to Player II is $-x^{\mathrm{T}}Ay$. We now have following two equivalent definitions of *solution* of the game G:

Definition 1. The triplet $(\bar{x}, \bar{y}, \bar{v}) \in S^m \times S^n \times R$ is called a *solution of the game G* if

$$\text{(i)}\ (\bar{x}^{\mathrm{T}}Ay) \geqslant \bar{v} \quad \text{for all } y \in S^n$$

and

$$\text{(ii)}\ x^{\mathrm{T}}A\bar{y} \leqslant \bar{v} \quad \text{for all } x \in S^m.$$

Here $\bar{x}$ (respectively, $\bar{y}$) is called the *optimal strategy* for Player I (respectively, Player II) and $\bar{v}$ is called the *value of the game G*.

[1] While preparing the revised draft of this paper, the authors came to know of a very recent Ref. [8] on fuzzy linear programming duality. The approach taken here is different from that of [8] as explained in Remark 3.