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Computers and Chemical Engineering 28 (2004) 2749–2763
Computers & Chemical Engineering
www.elsevier.com/locate/compchemeng

Analysis of linear programming in model predictive control

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Received 7 April 2004; received in revised form 17 August 2004; accepted 17 August 2004
Available online 1 October 2004

Abstract

The linear programming formulations of model predictive control are known to exhibit degenerate solution behavior. In this work, a multi-parametric linear programming technique is utilized to analyze the control laws that are generated from various linear programming based MPC routines. These various routines explore a number of factors, including objective function selection and constraint handling on the control laws generated from LP based MPC. A single input single output system is used to demonstrate that the use of input velocity penalties, input blocking, and ∞-norm objective functions can limit or eliminate this undesirable behavior. Finally, a paper machine cross directional control problem is used to demonstrate the control laws generated from LP based MPC for a multivariable example.

Keywords: Linear programming; Model predictive control; Analysis

1. Introduction

Model predictive control (MPC) is the de facto standard among control techniques for processes of high value. As opposed to other available control techniques, MPC provides the only methodology for handling process constraints in a manner that is consistent with the design and implementation of the controller (García, Prett, & Morari, 1989). Other advantages to using MPC include the scalability to a non-square multivariable framework, the ease of handling difficult process dynamics, and the ability to compensate for measured and unmeasured disturbances (Ogunnaike & Ray, 1994). Model predictive control utilizes a process model and results in an optimization problem that must be solved at each sampling period. The control law that is generated from MPC is open-loop optimal with respect to the corresponding objective function.

Traditionally, MPC algorithms minimize a quadratic objective, consisting of 2-norm measures for both tracking error and an input suppression term. For processes with constraints, DMC (Cutler & Ramaker, 1979) was developed to exploit quadratic programming (QP) to solve the optimization problem for each control move. For processes without constraints, a quadratic objective function leads to an analytical realization for the control law (García & Morshedi, 1986).

The earliest published formulations of MPC, however, used linear objective functions and linear programming (LP) solvers. Zadeh and Whalen (1962) first proposed the use of LP solvers for solving optimal control problems. A year later, Propoi (1963) proposed a technique involving on-line LP based optimal controllers and is widely credited with being the first conceived MPC algorithm. Other developments followed from these early papers on LP based MPC (LP-MPC). Richalet, Rault, Testud, and Papon (1977) note that linear programming, among other optimization techniques, can be used in the IDCOM approach. Chang and Seborg (1983) introduced an open-loop optimal LP control strategy that minimizes the absolute value, 1-norm, of the predicted error in a finite horizon. Morshedi et al.'s patent on dynamic process control (Morshedi, Cutler, Fitzpatrick, & Skrovanek, 1985) describes, in detail, a linear programming technique using a 1-norm criterion. In another paper, Morshedi, Cutler, and Skrovanek (1985) also note the use of a 1-norm criterion to

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0098-1354/$ – see front matter
doi:10.1016/j.compchemeng.2004.08.007

develop a predictive controller that can be used to minimize economic objectives. More recently, Megías, Serrano, and de Prada (2002) formulated a stable infinite-horizon model predictive controller that uses a 1-norm criterion for computational efficiency. Finally, both Gutman (1986) and Campo and Morari (1989) employ a min–max formulation with the ∞-norm for optimal level control. Campo and Morari (1989, 1990) also formulate a robust ∞-norm based MPC algorithm. Allwright and Papavasiliou (1991) recast Campo's formulation for the ∞-norm robust MPC algorithm into a more computationally efficient form.

In more recent publications (Dave, Willig, Kudva, Pekny, & Doyle III, 1997; Dave, Doyle III, & Pekny, 1999; Saffer II, Doyle III, Rigopoulos, & Wisnewski, 2001), Doyle III et al. describe the application of LP-MPC for large scale systems, notably the cross directional (CD) control of a paper machine (see, for example, Featherstone, Van Antwerp, and Braatz (2000) and references within). Paper machine CD control addresses the regulation of the important properties of paper in the machine, such as basis weight and moisture content, in the direction perpendicular to the direction of travel of the paper. More details of CD control are presented in Section 4.5. Previous work has considered customized solutions for the LP to bring the on-line solution time to within real-time implementation limits (Dave et al., 1997, 1999) and using efficient solvers and sparse matrix mathematics to perform the same calculations without the need to customize the LP solver (Saffer et al., 2001). Other researchers, including Backström, Gheorghe, Stewart, and Vyse (2001), Bartlett, Biegler, Backstrom, and Gopal (2002), Rigopoulos, Arkun, and Kayihan (1997), have described methods to solve constrained QP based MPC (QP-MPC) for the CD control problem. In the work of Rigopoulos et al. (1997) the problem size is first reduced by approximating the full problem by its main principal components and in Backström et al. (2001) the CD control problem is only used for one headbox of the machine, leaving the other headbox in their dual-headbox example to be handled as a measured disturbance. Finally Bartlett et al. (2002) use the structure of the paper machine problem and efficient programming formulation to increase the computational efficiency of a QP-MPC algorithm.

In a related effort, mixed integer linear programming (MILP) has been recently explored in a MPC framework. Bemporad, Borrelli, and Morari (2000, 2001) utilizes MILP for hybrid systems, which allowed for the inclusion of on/off type valves. Gatzke and Doyle III (2001) employ MILP based MPC to introduce soft constraints and prioritized control objectives into MPC control of a granulation process. Feather, Harrell, Liberman, and Doyle III (2003) applied an MILP based MPC formulation for hybrid systems that include the use of multiple models and operating heuristics. The current interest in LP-MPC applications, however, motivates a renewed analysis of some potentially problematic issues that have been described in the early variations of LP-based MPC. In Rao & Rawlings (2000), the authors note that LP based techniques for MPC can result in poor controller performance, namely idle and deadbeat behavior. The focus of the current study is the formal analysis of LP-MPC, and the development of controller tuning guidelines to overcome the issues associated with idle and deadbeat behavior. In the next section, the linear programming formulation of MPC is introduced. The section to follow uses the technique of multiparametric linear programming (mp-LP) to analyze, off-line, the control law that is generated from LP-MPC. Inspired by the works of Campo and Morari (1990), Hovd and Braatz (2001a,b), Meadows, Castro-Velez, Saffer II, and Doyle III (1999) and many others, the impact of particular terms in the objective function, the type of objective function that is to be used, and process constraints are then analyzed using the mp-LP solver. In related work, Hovd and Braatz (2001a,b) introduced an infinity-norm penalty as an additional term within a QP based MPC algorithm to minimize output and/or state constraint violations. This paper is focused on the analysis of LP based MPC (i.e., linear objectives only) using this technique as well as other well known objective function and constraint terms. Finally, a small multivariable paper machine CD control example is analyzed using the same technique to evaluate the presence of idle or deadbeat behaviors for multivariable control problems.

2. Model predictive control

As the name implies, MPC uses a model to predict the behavior of the process to determine the optimal control move. For this work, the model that is used within MPC is a discrete linear time-invariant state-space model:

$$\boldsymbol{x}_k = \boldsymbol{A}\boldsymbol{x}_{k-1} + \boldsymbol{B}\boldsymbol{u}_{k-1} \tag{1}$$

subject to the actuator and process constraints:

$$\boldsymbol{u}_{\min} \le \boldsymbol{u}_k \le \boldsymbol{u}_{\max}, \quad \Delta\boldsymbol{u}_{\min} \le \Delta\boldsymbol{u}_k \le \Delta\boldsymbol{u}_{\max}$$

and other given process constraints. The subscript k denotes the current time step and $k-1$ is the previous time step. The variables $\boldsymbol{x}$ and $\boldsymbol{u}$ represent the states and inputs, respectively. The $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices, assumed constant, are the usual state-space model terms. The addition of output constraints would be straightforward in the standard state space framework.

MPC requires the solution of a constrained control problem:

$$\min_{\boldsymbol{u}\{\boldsymbol{u}_k,\ldots,\boldsymbol{u}_{k+m-1}\}} \|\boldsymbol{P}\boldsymbol{x}_{k+p|k}\|_j^i + \Big\{\|\boldsymbol{Q}\boldsymbol{x}_{k+n|k}\|_j^i + \|\boldsymbol{R}_u\boldsymbol{u}_{k+n|k}\|_j^i + \|\boldsymbol{R}_{\Delta\boldsymbol{u}}\Delta\boldsymbol{u}_{k+n|k}\|_j^i\Big\},\ n = 1,\ldots,p \tag{2}$$

$$\text{s.t.}\quad \boldsymbol{x}_k = \boldsymbol{A}x_{k-1} + \boldsymbol{B}u_{k-1}, \qquad \boldsymbol{u}_{\min} \le \boldsymbol{u}_k \le \boldsymbol{u}_{\max},$$
$$\boldsymbol{u}_k = 0,\ k > m, \qquad \Delta\boldsymbol{u}_{\min} \le \Delta\boldsymbol{u}_k \le \Delta\boldsymbol{u}_{\max}$$

at each time step. The objective function consists of penalties on the states (with weights $\boldsymbol{P}$ for terminal state penal-