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Applied Mathematics Letters 18 (2005) 395–399
Applied Mathematics Letters
www.elsevier.com/locate/aml

A suggested approach for possibility and necessity dominance indices in stochastic fuzzy linear programming

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Received 1 January 2004; accepted 1 April 2004

Abstract

This paper presents a suggested approach for solving a stochastic fuzzy linear programming problem. This approach utilizes two possibility and two necessity dominance indices that have been introduced by Dubois and Prade [D. Dubois, H. Prade, Ranking fuzzy numbers in the setting of possibility theory, Information Sciences 30 (1983) 183–224]. The chance-constrained approach and the α-cut are used to transform the stochastic fuzzy problem to its deterministic-crisp equivalent, according to each of the four dominance indices. A numerical example is given.

Keywords: Possibility of dominance; Necessity of dominance; Chance-constrained approach

1. Introduction

Comparison of fuzzy numbers is considered one of the most important topics in fuzzy logic theory. The early and most important work in the field of comparing fuzzy numbers has been presented by Dubois and Prade [1]. A comparison between their work and other attempts that have been made in this area has been given by Bortolan and Degani [2]. On the other hand, the dominance possibility indices, which have been introduced by Dubois and Prade, were utilized in the field of fuzzy mathematical programming [3,4] and the field of stochastic fuzzy mathematical programming [5,6]. The approach used in these fields was based on formulating a possibility function, whether in the case of trapezoidal

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doi:10.1016/j.aml.2004.04.010

fuzzy numbers or the case of triangular fuzzy numbers. In this paper, we are going to utilize Dubois and Prade's dominance possibility and necessity indices, within a different approach, in the case of stochastic fuzzy linear programming problem. The dominance possibility and necessity, as well as the strict dominance possibility and necessity criteria, are utilized according to the chance-constrained method to transform the suggested problem to its deterministic-crisp equivalent. This approach helps avoiding any approximation that may exist due to comparing the inverse distribution function of fuzzy tolerance measures.

2. Model specification

In general, consider a stochastic fuzzy linear programming problem of the following form:

$$\text{Maximize} \qquad \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j \tag{1}$$

subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_i, \qquad i = 1, \ldots, m, \tag{2}$$

$$x_j \geq 0, \qquad j = 1, \ldots, n. \tag{3}$$

Here x_j, $j = 1, \ldots, n$ are non-negative decision variables, $\tilde{c}_j$, $j = 1, \ldots, n$ are fuzzy coefficients in the objective function, b_i, $i = 1, \ldots, m$ are independent random variables with known distribution functions, while $\tilde{a}_{ij}$ represents the fuzzy coefficient of the jth decision variable in the ith stochastic constraint. Thus, by incorporating fuzzy tolerance measures $\tilde{\delta}_i, 0 \leq \tilde{\delta}_i \leq 1, i = 1, \ldots, m$, and by utilizing the chance-constrained approach, the stochastic fuzzy constraints (2) can be transformed to their deterministic fuzzy equivalents as follows [5,6].

$$\Pr\left(\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_i\right) \geq \tilde{\delta}_i, \qquad i = 1, \ldots, m, \tag{4}$$

then,

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq F_i^{-1}(\tilde{\beta}_i), \qquad i = 1, \ldots, m, \tag{5}$$

where $\tilde{\beta}_i = 1 - \tilde{\delta}_i$, and $F_i^{-1}(.)$ is the inverse distribution function of the random variable $b_i, i = 1, \ldots, m$. It is apparent that this transformation requires the independent random variables to be continuous [6–8]. On the other hand, the deterministic fuzzy constraints set (5) is going to be represented by its crisp equivalent, according to each of the following four dominance indices that have been presented by Dubois and Prade [1]: Possibility of Dominance (PD), Possibility of Strict Dominance (PSD), Necessity of Dominance (ND), and Necessity of Strict Dominance (NSD). These indices for comparing fuzzy numbers are utilized whether $\tilde{a}_{ij}$ and $\tilde{\delta}_i$ are presented as trapezoidal or triangular fuzzy numbers.