Available online at www.sciencedirect.com
SCIENCE DIRECT®

Applied Energy 82 (2005) 40–63

APPLIED ENERGY

www.elsevier.com/locate/apenergy

An efficient linear programming model and optimization algorithm for trigeneration

Aiying Rong [*], Risto Lahdelma

Department of Information Technology, University of Turku, Lemminkäisenkatu 14 A, FIN-20520 Turku, Finland

Received 24 May 2004; revised 20 July 2004; accepted 31 July 2004
Available online 15 December 2004

Abstract

Trigeneration is a booming technology for efficient and clean provision of energy. It has potential for reducing pollution emissions dramatically. Similar to combined heat and power (CHP) production, cost-efficient operation of a trigeneration system can be planned using an optimization model based on hourly load forecasts. A long-term planning model decomposes into thousands of hourly models, which can be solved separately. In this paper, we model the hourly trigeneration problem as a linear programming (LP) model with a joint characteristic for three energy components to minimize simultaneously the production and purchase costs of three energy components, as well as CO_2 emissions costs. Then we explore the structure of the problem and propose the specialized *Tri-Commodity Simplex* (TCS) algorithm that employs this structure efficiently. The speed of TCS is based on extremely fast basis inverse operations and reuse of old basic solutions from previously solved hourly models. We compare the performance of TCS with realistic models against an efficient sparse Simplex code using the product form of inverse. In test runs, TCS is from 36 to 58 times faster when starting from the initial basis and from 43 to 179 times faster when reusing the old basis.

Keywords: Linear programming; Trigeneration; Energy; Optimization; CO_2 emissions

---

[*] Corresponding author.
E-mail address: aiying@it.utu.fi (A. Rong).

0306-2619/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.apenergy.2004.07.013

Nomenclature

Abbreviations

CHP	Combined Heat and Power
LP	Linear Programming
TCS	Tri-Commodity Simplex

Indices

j	subscript of extreme point or decision variable
p, q, r	super/subscripts or prefixes for the three energy products in trigeneration
u	subscript of plant
$u(p)$, $u(q)$, $u(r)$	the plant corresponding to basic variable on the p-, q- or r-balance

Symbols (matrices in bold-face, others in italics)

$\mathbf{A}$	constraint matrix of the LP model
b	right-hand-side vector of the LP model
$\mathbf{B}$	non-singular basis matrix
c, c^{B}, c^{N}	row vector of cost coefficients of the LP model and its basic and non-basic part
c_1^{B}, c_2^{B}	partition of c^{B} according to basis structure
c^{p-}, c^{q-}, c^{r-}	penalty costs for slack in the energy balances
c^{p+}, c^{q+}, c^{r+}	penalty costs for surplus in the energy balances
c_j, p_j, q_j, r_j	extreme point of the characteristic operating region of a plant in terms of cost and the three energy products
d	row vector of reduced costs of non-basic variables
d_j	reduced cost of variable x_j
$\mathbf{D}$	square submatrix of $\mathbf{B}$
$\mathbf{E}$	submatrix of $\mathbf{B}$
$\mathbf{I}$	identity matrix
J	index set of extreme points of the operating regions of all plants
J_u	index set of extreme points in plant u ($J = \cup_{u \in \mathrm{U}} J_u$)
m	number of constraints in the LP model
n	number of variables in the LP model
$\mathbf{N}$	non-basic part of $\mathbf{A}$
$\mathbf{N}_{\cdot j}$, $\mathbf{N}_{\cdot j1}$, $\mathbf{N}_{\cdot j2}$	column j of $\mathbf{N}$ and its partition according to basis structure
$p^{(i)}$, $q^{(i)}$, $r^{(i)}$	p-, q- and r- coefficients in $\mathbf{D}$, $i = 1,\ldots,6$
P, Q, R	demand for p-, q- and r-energy products
U	index set of plants (trigeneration plants, CHP plants, separate energy sources)
x, x^{B}, x^{N}	vector of decision variables and its partition into basic and non-basic variables
x_j	decision variables indicating the operating level of each plant in terms of the extreme points of the operating region, $j \in J_u$.