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# Effective securities in arbitrage-free markets with bid–ask spreads at liquidation: a linear programming characterization

Mariagiovanna Baccara<sup>a</sup>, Anna Battauz<sup>b</sup>, Fulvio Ortu<sup>c,\*</sup>

<sup>a</sup>*Stern School of Business, New York University, USA*

<sup>b</sup>*Istituto di Metodi Quantitativi, Università Bocconi, Italy*

<sup>c</sup>*Istituto di Metodi Quantitativi and IGIER, Università Bocconi, Viale Isonzo 25, Milano 20135, Italy*

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## Abstract

We consider a securities market with bid–ask spreads at any period, including liquidation. Although the minimum-cost super-replication problem is non-linear, we introduce an auxiliary problem that allows us to characterize no-arbitrage via linear programming techniques. We introduce the notion of effective new security and show that effectiveness restricts the no-arbitrage bid and ask prices of a new security to the interval defined by the minimum-cost problem. We discuss in detail the cases in which the boundaries of this interval can be reached without violating no-arbitrage.

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*Keywords:* Arbitrage; Bid–ask prices; Linear programming; Effective securities

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## 1. Introduction

The valuation of securities via super-replication in the presence of market frictions and its interplay with no-arbitrage is one of the most active research areas in finance theory. The topic has been analyzed both in discrete-time, starting from [Bensaid](#)

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\*Corresponding author.

*E-mail address:* [fulvio.ortu@unibocconi.it](mailto:fulvio.ortu@unibocconi.it) (F. Ortu).

et al. (1992) and Jouini and Kallal (1995), and in continuous-time, dating back to Cvitanic and Karatzas (1993).<sup>1</sup>

This paper follows the discrete-time, event-tree approach and offers two main contributions. First, we show how to employ linear programming techniques to characterize no-arbitrage in markets with bid–ask spreads. With respect to the existing literature the contribution is that our linear programming approach *works also with bid–ask spreads at liquidation*. Second, we supply a linear programming-based proof of the fact that no-arbitrage per se imposes only an upper bound on the bid and a lower bound on the ask price of a new security. We then introduce the notion of *effective new security* and show that this notion characterizes the new securities whose bid–ask spreads are bounded.

In a seminal paper, Bensaid et al. (1992) incorporate bid–ask spreads in the standard binomial option pricing model and solve the super-replication problem via dynamic programming. Still in a binomial model but without bid–ask spreads at liquidation, Edirisinghe et al. (1993) show how to reformulate the super-replication problem as a linear programming one. Naik (1995) and Ortu (2001) analyze the general event-tree framework without bid–ask spreads at liquidation and use the linearized super-replication problem and its dual to provide alternative characterizations of no-arbitrage.

In this paper we address the general event-tree framework with bid–ask spreads also at liquidation. The presence of bid–ask spreads at liquidation arises in many practical applications. A European call option, for instance, is typically settled at maturity either with delivery of the underlying, or by cash, or at the discretion of the short position. In a world without bid–ask spreads at maturity, these different types of settlement are payoff-equivalent. In actual markets, however, bid–ask spreads are present also at maturity and different settlement provisions produce different payoff profiles. Bid–ask spreads at liquidation introduce a non-linearity in the otherwise linear super-replication problem. Indeed, investors typically aggregate their long and short positions with the same broker. This implies that rather than the cumulative long and short positions separately, what matters at the moment of final liquidation are the net positions held in each security. With bid–ask spreads at liquidation, this makes the terminal payoff, and hence the super-replication problem, non-linear in the intertemporal trading strategies.

To deal with this non-linearity, we construct an auxiliary linear program with the same value function as the original problem, such that any solution to the super-replication problem is a linear transformation of a solution to the auxiliary linear program. To construct this auxiliary program we first partition the set of feasible trading strategies according to the sign of the net positions at liquidation. Then,

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<sup>1</sup>Other contributions related to the present work are Dermody and Rockafellar (1991), Chen (1995), Chateaufneuf et al. (1996), Milne and Neave (1997), Charupat and Prisman (1997), Jouini (2000), Koehl et al. (2001), Zhang et al. (2002), Huang (2002), Delbaen et al. (2002). We refer to Cvitanic (2001) for a detailed survey of the literature on super-replication without and with frictions in continuous-time models.

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