

# A class of multiobjective linear programming model with fuzzy random coefficients

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## Abstract

The aim of this paper is to deal with a multiobjective linear programming problem with fuzzy random coefficients. Some crisp equivalent models are presented and a traditional algorithm based on an interactive fuzzy satisfying method is proposed to obtain the decision maker's satisfying solution. In addition, the technique of fuzzy random simulation is adopted to handle general fuzzy random objective functions and fuzzy random constraints which are usually hard to be converted into their crisp equivalents. Furthermore, combined with the techniques of fuzzy random simulation, a genetic algorithm using the compromise approach is designed for solving a fuzzy random multiobjective programming problem. Finally, illustrative examples are given in order to show the application of the proposed models and algorithms.

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## 1. Introduction

Among types of uncertainty surrounding real life problems, randomness (stochastic variation) and fuzziness (vagueness) play a pivotal role. Accordingly, stochastic programming and fuzzy programming have been proposed to make decisions under an uncertainty environment. Different types of stochastic programming and fuzzy programming models have been developed to suit the different purposes of management, such as the expectation model [1], chance constrained programming [2,3], the minimum risk problem [4], the modality approach and the fractile approach [5] etc. In these models, randomness and fuzziness are considered as separate aspects. However, in a decision-making process, we may face a hybrid uncertain environment where fuzziness and randomness coexist. In such cases, the concept of fuzzy random variable introduced by Kwakernaak [6] is a useful tool dealing with the two types of uncertainty simultaneously.

Recently, several researchers have considered the issue of combining fuzziness and randomness in an optimization framework such as Wang and Qiao [7,8], Luhandjula [9,10], Katagiri et al. [11], and Liu [3,12,1,4]. In [7,8], Wang and Qiao discussed the distribution problems for linear programming with fuzzy random coefficients. In [9], Luhandjula

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employed a semi-infinite approach in order to convert the original fuzzy random linear programming problem into a stochastic programming one so that the techniques of stochastic optimization can apply. In [10], Luhandjula proposed a unifying methodological approach to transform the constraints with fuzzy random coefficients into crisp constraints. In [11], Katagiri et al. proposed an interactive satisfying method to solve the fuzzy random multiobjective 0-1 programming problem. Besides, in [3,12] Liu presented fuzzy random chance-constrained programming, fuzzy random dependent-chance programming, and designed some hybrid intelligent algorithms in order to solve them effectively. In [1,4], Liu and Liu presented an expected value model and minimum risk problem for the fuzzy random multiobjective programming problem and designed some hybrid intelligent algorithms.

The purpose of this paper is to present two approaches of solving multiobjective linear programming with fuzzy random coefficients. Our research is based on the chance measure of fuzzy random events [3]. This paper is organized as follows. Section 2 recalls some definitions and results about fuzzy random variables. Section 3 studies the prob-pos constrained multiobjective programming model. A crisp equivalent model is proposed for a special type of fuzzy random variables, and an interactive fuzzy satisfying method is adopted to obtain the decision maker's satisfactory solution. Section 4 considers the prob-nec constrained multiobjective programming model. Fuzzy random simulation and fuzzy random simulation-based genetic algorithm using compromise approach are presented in Sections 5 and 6, respectively. Finally, illustrative examples are given in order to show the application of the proposed models and algorithms. The results show that the fuzzy random simulation-based genetic algorithm is effective.

## 2. Fuzzy random variable

Fuzzy random variable, which was introduced by Kwakernaak [6] in 1978, is a concept to depict the phenomena in which randomness and fuzziness appear simultaneously. Since then, its variants and extensions were presented by other researchers, e.g., Colubi et al. [13], Kruse and Meyer [14], López-Díaz and Gil [15], Puri and Ralescu [16] and Liu and Liu [17].

In this paper, the definition of fuzzy random variable and the results are cited from [17].

Let  $\mathcal{F}$  be a collection of fuzzy variables defined on the possibility space.

**Definition 1.** Let  $(\Omega, \mathcal{A}, \text{Pr})$  be a probability space. A fuzzy random variable is a function  $\xi : \Omega \rightarrow \mathcal{F}$  such that for any Borel set  $B$  of  $R$ ,

$$\xi^*(B)(w) = \text{Pos}\{\xi(w) \in B\}$$

is a measurable function of  $w$ .

**Lemma 1.** Let  $f : R^n \rightarrow R$  be a continuous function and  $\xi_i$  be fuzzy random variables defined on  $(\Omega_i, \mathcal{A}_i, \text{Pr}_i)$ ,  $i = 1, 2, \dots, n$ , respectively. Then  $f(\xi) = f(\xi_1, \xi_2, \dots, \xi_n)$  is a fuzzy random variable on the product probability space  $(\Omega_1 \times \Omega_2 \times \dots \times \Omega_n, \mathcal{A}_1 \times \mathcal{A}_2 \times \dots \times \mathcal{A}_n, \text{Pr}_1 \times \text{Pr}_2 \times \dots \times \text{Pr}_n)$ , defined by

$$f(\xi)(w_1, w_2, \dots, w_n) = f(\xi_1(w_1), \xi_2(w_2), \dots, \xi_n(w_n))$$

for all  $(w_1, w_2, \dots, w_n) \in \Omega_1 \times \Omega_2 \times \dots \times \Omega_n$ .

**Lemma 2.** Assume that  $\xi$  is a fuzzy random vector, i.e., with the  $n$ -tuple of fuzzy random variables  $(\xi_1, \xi_2, \dots, \xi_n)$ , and  $g_r$  are real-valued continuous functions for  $r = 1, 2, \dots, p$ . Then

- (i) the possibility  $\text{Pos}\{g_r(\xi(w)) \leq 0, r = 1, 2, \dots, p\}$  is a random variable;
- (ii) the necessity  $\text{Nec}\{g_r(\xi(w)) \leq 0, r = 1, 2, \dots, p\}$  is a random variable.

Consider the following multiobjective programming problem with fuzzy random coefficients

$$\begin{aligned} & \max\{f_1(x, \xi), f_2(x, \xi), \dots, f_m(x, \xi)\} \\ & \text{s.t. } g_r(x, \xi) \leq 0, \quad r = 1, 2, \dots, p, \end{aligned} \tag{2.1}$$

where  $x$  is a  $n$ -dimensional decision vector,  $\xi = (\xi_1, \xi_2, \dots, \xi_n)$  is a fuzzy random vector,  $f_i(x, \xi)$  are objective functions,  $i = 1, 2, \dots, m$  and  $g_r(x, \xi)$  are constraint functions,  $r = 1, 2, \dots, p$ . Because of the existence of fuzzy random vector  $\xi$ , problem (2.1) is not well-defined. That is, the meaning of maximizing  $f_i(x, \xi)$ ,  $i = 1, 2, \dots, m$  is not clear and the constraints  $g_r(x, \xi) \leq 0$ ,  $r = 1, 2, \dots, p$  do not define a deterministic feasible set.

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