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Interactive compensatory fuzzy programming for decentralized multi-level linear programming (DMLLP) problems

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Abstract

This paper presents interactive compensatory fuzzy programming for decentralized multi-level linear programming (DMLLP) problems. By adjusting the cooperative decision making process between the different levels and also between the decision makers of the same level; our aim is to obtain a preferred compensatory compromise Pareto-optimal solution for DMLLP. For this, the weights of objectives at each level are assigned by the next upper level decision maker (DM) by using analytic hierarchy process (AHP) or any other weighting methods. The weight of any objective for whole system is equal to the product of the weights on the path tying it to the top decision maker DM_0. Using these weights, equivalence is established such that the satisfactory levels of all objectives are proportional to their own weights. Werners' compensatory "fuzzy and" operator is offered to solve DMLLP problem. The most important idea to be emphasized is that equivalence is established such that the satisfactory levels of all objectives are proportional to their own weights. Thanks to this equivalence, DMLLP problem has been transformed to the multi-objective linear programming (MOLP) problem at level 0, the equivalence is reflected to the compensatory model within the constraints, and the equivalence also enables all DMs to obtain proportional satisfactions with their weights as much as possible. So, in our compensatory model, a reduction on equivalent satisfactory level of one DM can be compensated for by an increase in the equivalent satisfactory level of another DM. Furthermore, being developed a finite interactive iterative procedure with maximum interaction step, a set of compensatory solutions which are also Pareto-optimal is obtained, depending on compensation parameter γ. Giving a theorem, we will show that the solutions generated by Werners' compensatory "fuzzy and" operator do guarantee Pareto-optimality for our DMLLP problem. And comparing it with some other computational efficient compensatory fuzzy aggregation operators we will conclude that this operator is more appropriate for DMLLP. Illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed interactive fuzzy compensatory method for DMLLP.

Keywords: Compensatory operators; Multi-level programming; Fuzzy mathematical programming; Fuzzy decision making

1. Introduction

Multiple level programming problems are frequently encountered hierarchical organizations of large companies or the decentralized systems of nonprofit and government organizations. Multiple level programming techniques explicitly assign each decision maker (DM) a unique objective, a set of decision variables, and a set of common constraints that

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affect all DMs. These problems have two interpretations dependent on whether there is a cooperative relationship among DMs or not.

The Stackelberg solution has been usually employed as a solution concept to multi-level programming problems [4–6,2,3,7,8,27,32,34]. For example, in the bi-level programming problems, a decision maker at the upper level makes a decision subject to an optimization problem for a DM at the lower level. When the Stackelberg game [27]—a special case of a two-person, non-zero sum, non-cooperative game, with full information—is employed, it is assumed that there is no communication between the two DMs, or they do not make any binding agreement even if such communication exists. In the Stackelberg game, each DM completely knows objective functions and constraints of the two DMs, and the DM at the upper level (leader) first make a decision and then the DM at the lower level (follower) specifies a decision so as to optimize an objective function with full knowledge of the decision of the leader. But, it is known that the problem for obtaining the Stackelberg solution is a non-convex programming problem with special, even if the objective functions of both DMs and the common constraint functions are linear. And also, in general, the Stackelberg solution does not satisfy Pareto optimality because of its non-cooperative nature. So, the multi-level programming problems are difficult to solve and have been proved to be strongly NP- hard [26]. Some existing numerical techniques such as the vertex enumeration approach, the extreme point search approach, the procedure based on the Karush–Kuhn–Tucker condition, the penalty function approach, and the descent methods are effective only for solving very simple problems [4–8,30,31,34].

However, concerning the hierarchical decision problem in the decentralized firm, it is quite natural that decision makers are regarded as to be cooperative rather than to be completely non-cooperative. To overcome the above mentioned difficulties, in 1996, Lai [12] and Shih et al. [24] have proposed a new solution concept, which is different from the concept of a Stackelberg solution, for problems such that decisions of DMs in all the levels are sequential and all the DMs essentially cooperate with each other. In their papers, they developed a fairly effective fuzzy approach by using the concept of tolerance membership functions and multi-objective decision making. The idea is to use the basic fuzziness and vagueness nature of such large hierarchy systems to make the complexity tractable. Generally, in their approaches, the upper level defines his or her tolerances by the use of membership function which constraints the lower level DMs' feasible space. The resulting iterative procedure, instead of the usually used extreme point search, relies on the change of membership functions which expresses the degree of satisfaction of the solutions to both the upper- and the lower-level DMs. So, their approaches explore the inherent vagueness of the system and thus generate no significant additional constraints. In fact they significantly reduce the amount of computation required for large multiple level decentralized programming problems. Unfortunately, there is a possibility that their method leads a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and the decision variables. To avoid such problems in the methods of Lai [12] and Shih et al. [24], by eliminating the fuzzy goals for decision variables, Sakawa et al. [21] have developed interactive fuzzy programming for two-level linear programming problems. Moreover, from the viewpoint of experts' imprecise or fuzzy understanding of the nature of parameters in the problem formulation process [16], they have extended it to interactive fuzzy programming for two-level linear programming problems with fuzzy parameters [22]. These results have been extended to deal with two-level linear fractional programming problems both without and with fuzzy parameters [17,23]. Along these results, as additional results, extensions to multi-level 0–1 programming problems, two-level nonconvex programming problems have been done through genetic algorithms [20,19]. Sakawa and Nishizaki [18] present an interactive fuzzy method, which consists of two phases, for decentralized two-level linear programming problems. In his paper, Sinha [28] has suggested fuzzy mathematical programming which is a supervised search procedure (supervised by the higher level decision maker). The higher level DM provides the preferred values of decision variables under his control (to enable the lower level DM to search for his optimum in a wider feasible space) and the bounds of his objective function (to direct the lower level DM to search for his solutions in the right direction). Recently, Ahlatcioglu and Tiryaki [1] presented two interactive fuzzy programming approaches for a decentralized two-level linear fractional programming (DTLLFP) problem with a single decision maker DM_0 at the upper level and multiple DMs at the lower level. In both their approaches, with the AHP method, DM_0 assigns weights to objectives at the lower level. An equivalence is established such that the satisfactory levels of all objectives are proportional to their own weights. Transformed main problems are constructed corresponding to DTLLFP problem. If the DM_0 is not satisfied with this solution, a strongly efficient satisfactory solution can be reached by interacting with him or her.

The fuzzy approaches mentioned above explore the inherent vagueness of the system and thus generates no significant additional constraints. In fact they significantly reduce the amount of computation required for large multiple level