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Shakedown analysis using the p-adaptive finite element method and linear programming

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Abstract

In this paper, we investigate the use of an adaptive p-version of the finite element method to perform shakedown analyses of 2D plane strain problems within the static approach. Moreover, efficient piecewise linearizations of the yield surfaces are carried out in a semi-adaptive fashion so that we need to solve the more tractable linear, rather than nonlinear, programming problems. State-of-the-art linear programming solvers, based on the very efficient interior point methodology, are used for solving the optimization problems. Various numerical examples are provided to compare the efficiency of the proposed approach with those of uniform and nonuniform p-mesh models and nonlinear programming.

Keywords: Shakedown analysis; Limit analysis; Adaptive p-elements; Plasticity; Linear programming

1. Introduction

The use of simplified methods is an appealing approach to obtain some essential information, such as the load-carrying capacity of structural systems, for use in preliminary design or safety evaluation. Such, so-called “direct”, schemes avoid computationally expensive step-by-step analyses that trace inelastic structural responses to a given loading history. One such method is limit analysis which is used to estimate the collapse load of perfectly-plastic media under a monotonically increasing load regime. A shakedown analysis, a generalization of limit analysis, is also commonly used when the external loading, as is often the case, is repeated in nature with its precise history being unknown except for its upper and lower limits. Performing efficient and accurate shakedown analyses constitutes the focus of the present paper.

The occurrence of shakedown implies that dissipative, yielding processes eventually cease, while its obverse leads to failure by either incremental collapse (characterized by unbounded deformation growth for each cycle of loading) or alternating plasticity (eventually leading to fractures by a low cycle fatigue type phenomenon). For an overview, the interested reader is referred, for instance, to the still useful proceedings of the 1977 NATO conference held at Waterloo [1], to the monograph by König [2], to such classical papers as Belytschko [3] and Maier [4], and to the key survey articles by König and Maier [5], Maier et al. [6] and Maier [7]. Worthy of mention are some more recent works, and the numerous references contained therein, such as those by Chen and Ponter [8], Staat and Heitzer [9], and Vu et al. [10].

As discussed in [11], two main difficulties, which are in fact typical of computational plasticity problems in general, are encountered when computing limit and shakedown loads. Firstly, in plane strain and 3D problems with certain yield conditions such as von Mises, volumetric or isochoric locking may occur. Secondly, practically motivated structures often lead to large numerical models that can become computationally infeasible or even intractable. Tin-Loi and Ngo [11] explored the use of the p-version of the finite element method (FEM) for overcoming locking when carrying out limit analyses. This method was found to be robust and accurate, and was subsequently extended to solve shakedown problems as well [12]. However, as pointed out in these papers, whilst increasingly more accurate estimates of both collapse and shakedown limits are obtained as the degree of polynomial p is raised, simply increasing p uniformly does incur a computing cost for high order elements, primarily in the effort required

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to solve the large size nonlinear programming (NLP) problems that arise.

An attempt to reduce computational costs was explored recently by Ngo and Tin-Loi [13] who used a p-adaptive scheme. Despite the fact that plasticity was involved, the adaptive process was carried out in the preoptimization (elastic) stage only. The uniform p-version FEM approach was modified, as is conventional [14–16], by using some error estimators to evolve the original FE mesh into a nonuniform one for which the number of degrees of freedom (DOFs) is much less than in the similar (i.e. of the same geometrical discretization) p-uniform mesh. Whilst reduction in the size of the problem showed a significant gain in computational time in the nonlinear optimization solution – the most time consuming phase of solving limit and shakedown problems – it was concluded that further computational efficiency was desirable.

In the present paper, we investigate use of piecewise linear (PWL) yield surfaces in concert with the p-adaptive FEM. The primary reason for this approach is to take advantage of the fact that linear programming (LP) problems are far easier to solve than NLP problems (see e.g. [17,18]), especially with the ready availability of state-of-the-art interior point LP solvers, as provided, for instance, via the GAMS modeling system [19] which is available freely through the NEOS server over the internet [20].

The advantages of PWL yield surfaces, leading to vastly improved computational efficiency and a mathematical structure more susceptible for theoretical developments, have been widely recognized since the 1970s (see e.g. [1,4]). Ad hoc [17,21,22] as well as automatic methods [23–25] for piecewise linearizing yield surfaces have been proposed in the literature. It should be noted that the latter schemes are not necessarily more efficient. For instance, Cannarozzi's [24] nontraditional linearizing procedure appears to be attractive at first sight. It iteratively approaches the plastic admissibility domain through a sequence of circumscribing PWL yield polyhedra, with each polyhedron representing a better approximation to the actual yield domain than the preceding one. The procedure is theoretically capable of evaluating the limit load as accurately as required, if it is implemented with a complete search for points in the structure where the yield condition is violated. However, such a search can become very expensive, and may also require too many associated optimization runs (i.e. one run per linearization). In this paper, the adaptive scheme of Tin-Loi [25] is used. Basically, starting from a coarse discretization, the PWL surface is subsequently refined locally from information provided by the initial run. This local refinement has the effect of not only improving the solution accuracy but also keeping the LP problem size as small as possible.

The organization of this paper is as follows. In Section 2, we present some fundamental notions related to the shakedown problem within our adopted static approach, including its appropriate discretization by the p-version FEM. The adaptive scheme we use to evolve a uniform mesh into a nonuniform one at the elastic stage is also briefly presented. Section 3 deals with the construction of PWL yield surfaces using a heuristic, semi-adaptive approach. Some 2D numerical examples involving the limit and shakedown analyses of plane strain structures obeying von Mises yield criterion are presented in Section 4. This class of structures has been chosen in view of its propensity to lock when modeled using conventional displacement h-elements. We conclude in Section 5.

2. Preliminaries

In this section, we briefly review the shakedown analysis problem using the static approach. Incidentally, the use of a static approach is dictated primarily by the simplicity of implementation, and it also avoids the possible difficulties of having to minimize the typically nonsmooth objective function [26] that arises in a kinematic formulation. The discrete shakedown formulation by the p-version FEM is then briefly presented, followed by an outline of our p-adaptive scheme applied to the elastic phase. Classical conditions are assumed, namely, small displacement gradients and hence linear kinematic relations, quasistatic loading, and an elastic perfectly-plastic material that is stable in Drucker's sense.

2.1. Static shakedown approach

The static approach, based on Melan's static shakedown theorem, for calculating the shakedown multiplier μ_{SD} is briefly described in the following; for details, a standard reference such as König [2] or Morelle [27] should be consulted.

Consider a body with a bounded domain V subjected to a variable loading $(\mathbf{g}, \mathbf{q})$, where $\mathbf{g}$ and $\mathbf{q}$ are the body and surface forces in V and on C, respectively. Its boundary $\partial V = C \cup S$ consists of a fixed region S, where the displacement rates $\mathbf{u} = \mathbf{0}$, and of a free and possibly loaded part C (on which any surface forces $\mathbf{q}$ are applied). For simplicity, the prescribed displacements on S are assumed to be zero. The cyclic loading $(\mathbf{g}, \mathbf{q})$ can vary arbitrarily with time within a given load domain $\mathcal{L}$. Denoting by $\boldsymbol{\sigma}$ and $\boldsymbol{\epsilon}$, respectively, the stress and strain fields in the body, the following equilibrium equations and static boundary conditions must be satisfied:

$$\begin{aligned} -\nabla \cdot \boldsymbol{\sigma} + \mathbf{g} = \mathbf{0} \quad & \text{in } V, \\ \mathbf{n} \cdot \boldsymbol{\sigma} = \mathbf{q} \quad & \text{on } C, \end{aligned} \tag{1}$$

where $\mathbf{n}$ is the unit normal vector at points on C.

For the same body (V, S, C) and the same loading $(\mathbf{g}, \mathbf{q})$, the purely elastic stress $\boldsymbol{\sigma}^e$ and strain $\boldsymbol{\epsilon}^e$ fields obey the classical relation

$$\boldsymbol{\sigma}^e = \mathbf{D}\boldsymbol{\varepsilon}^e, \tag{2}$$

where $\mathbf{D}$ is the familiar material constitutive matrix. Any residual stress field $\boldsymbol{\rho}$ can also be defined such that it satisfies (1) with $\mathbf{g} = \mathbf{0}$ and $\mathbf{q} = \mathbf{0}$.

We now assume that the yield condition is of the form

$$\boldsymbol{\sigma} \in \mathcal{K}, \tag{3}$$