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Robust controller design by linear programming with application to a double-axis positioning system

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Abstract

A linear programming approach is proposed to tune fixed-order linearly parameterized controllers for stable LTI plants. The method is based on the shaping of the open-loop transfer function in the Nyquist diagram. A lower bound on the crossover frequency and a new linear stability margin which guarantees lower bounds for the classical robustness margins are defined. Two optimization problems are proposed and solved by linear programming. In the first one the robustness margin is maximized for a given lower bound on the crossover frequency, whereas in the second one the closed-loop performance in terms of the load disturbance rejection is optimized with constraints on the new stability margin. The method can directly consider multi-model as well as frequency-domain uncertainties. An application to a high-precision double-axis positioning system illustrates the effectiveness of the proposed approach.

Keywords: Linear programming; Convex optimization; PID controller; Robustness margin

1. Introduction

Many controller design methods are based on optimization techniques. In early works, the main interest was to find an analytical solution to the optimization problem. Recently, with new progress in numerical methods to solve convex optimization problems, new approaches for controller design with convex objectives and constraints have been developed. These methods usually lead to controllers of at least the same order as the plant model. However, design of restricted-order controllers (like PID controller) leads to non-convex optimization problems in the controller parameter space.

Nowadays, PID controllers are still extensively used in industrial applications and a lot of methods have been proposed in the literature to simply tune the controller parameters. H_∞ optimization of fixed-order controllers have been the subject of many research works (Malan et al., 1994; Grigoriadis & Skelton, 1996) which adopt nonlinear and non-convex algorithms. These methods require generally heavy computations and do not guarantee the global optimum to be achieved. Moreover, it is generally difficult to tune the weighting filters automatically. Industrial users prefer classical specifications like gain and phase margins, crossover frequency, maximum of the sensitivity and complementary sensitivity functions and good set point and disturbance rejection responses. A combination of time and frequency-domain specifications makes the optimization problem more complicated. A model-based method for optimizing the parameters of a PID controller with a frequency-domain criterion and closed-loop specifications is proposed in Harris and Mellichamp (1985).

The objective function is minimized with a version of the simplex method. Schei (1994) proposes a non-convex constrained optimization method to maximize the controller gain in low frequencies with constraints on the maximum of the sensitivity and complementary sensitivity functions. Åström et al. (1998) show that the maximum of the sensitivity function is an appropriate design variable and together with optimization of load disturbance rejection and

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good choice of set point weight can give generally very high performances for PI controllers. An algorithm to find a local minimum of the criterion is proposed. Panagopoulos et al. (2002) extend this method to design PID controllers. A numerical solution to this non-convex problem is also developed in Hwang and Hsiao (2002).

Convex optimization approaches to fixed-order controller design are rather limited. In Grassi and Tsakalis (1996) a convex optimization method for PID controller tuning by open-loop shaping in frequency domain is proposed. The infinity-norm of the difference between the desired open-loop transfer function and the achieved one is minimized. This method, however, needs the desired open-loop transfer function to be defined and it cannot be applied to the case of multi-model uncertainty. Ho et al. (1997) show that all stabilizing PID controllers with fixed proportional gain can be found by resolving a linear programming problem for the derivative and integral gains. Blanchini et al. (2004) go further by showing that, given the value of the proportional gain, the region of the plane defined by the derivative and integral gains, where a considered H_∞ constraint is satisfied, consists of the union of disjoint convex sets. However, both methods have the drawback of fixing the proportional gain a priori. In Keel and Bhattacharyya (1997) a PID controller design based on linear programming for pole placement and model matching problem is proposed. A serious difficulty in this approach is to specify the desired closed-loop poles and desired closed-loop transfer function. Recently, a new approach based on the generalized Kalman–Yakoubovic–Popov lemma has been proposed to tune the linearly parameterized controllers in the Nyquist diagram (Hara et al., 2006). The idea is to define several convex regions in the complex plane and design the controller such that in each frequency interval the open-loop transfer function lies in one of the regions. However, this method seems to be too complex for industrial applications.

In this paper, a loop shaping method in the Nyquist diagram is proposed. A new stability margin is defined which guarantees a lower bound for the gain, phase and modulus margins (the inverse of the maximum of the sensitivity function). The main property of the new margin is that a constraint on this margin is linear with respect to the parameters of linearly parameterized controllers. Therefore, optimizing load disturbance rejection with constraint on this margin leads to a linear constrained optimization problem which can be solved by linear programming. A lower bound for the crossover frequency is also defined which also leads to a linear constraint for the optimization problem. With this constraint, an optimization problem to maximize the new robustness margin can be solved by linear programming. The method can be applied to stable plants represented by transfer functions with pure time delay or simply by non-parametric models in the frequency domain. The robustness of the closed-loop system with respect to unmodeled dynamics is ensured with the constraint on the new linear margin, while the multi-model uncertainty can be considered easily by increasing the number of constraints. The proposed method can be used for PID controllers as well as higher-order linearly parameterized controllers in discrete or continuous time. Standard linear optimization solvers can be used to find the global optimal controller. The proposed method is applied to a double-axis linear permanent magnet synchronous motor (LPMSM). This type of motors can be used for wafer fabrication and inspection and other high precision positioning systems. A generalized synchronization control for such systems has been proposed in Xiao et al. (2005).

This paper is organized as follows: in Section 2 the class of models, controllers and the control objectives are defined. Section 3 introduces a new linear stability margin, a lower bound on the crossover frequency and presents the linear optimization problem. Simulation results are given in Section 4 and experimental results in Section 5. Finally, Section 6 gives some concluding remarks.

2. Problem formulation

2.1. Plant model

The class of linear time-invariant SISO systems with no pole in the right half plane is considered. It is assumed that a set of non-parametric models in the frequency domain is available. This set can be obtained either by spectral analysis from several identification experiments at different operating points or from parametric models in the form of rational transfer functions with pure time delay. Suppose that the dynamics of the system can be captured by a sufficiently large finite number of frequency points N. The number of models in the set is m and so the model set can be presented by

$$\mathscr{M} \triangleq \{G_i(\mathrm{j}\omega_k) | i = 1, \ldots, m; \quad k = 1, \ldots, N\}. \tag{1}$$

2.2. Controller parameterization

The class of linearly parameterized controllers is considered:

$$K(s) = \rho^{\mathrm{T}} \phi(s), \tag{2}$$

where

$$\rho^{\mathrm{T}} = [\rho_1, \rho_2, \ldots, \rho_n], \tag{3}$$

$$\phi^{\mathrm{T}}(s) = [\phi_1(s), \phi_2(s), \ldots, \phi_n(s)], \tag{4}$$

n is the number of controller parameters and $\phi_i(s), i = 1, \ldots, n$, are transfer functions with no RHP pole. With this parameterization, every point on the Nyquist diagram of $K(\mathrm{j}\omega)G_i(\mathrm{j}\omega)$ can be written as a linear function of the controller parameters ρ:

$$\begin{aligned} K(\mathrm{j}\omega_k)G_i(\mathrm{j}\omega_k) &= \rho^{\mathrm{T}}\phi(\mathrm{j}\omega_k)G_i(\mathrm{j}\omega_k) \\ &= \rho^{\mathrm{T}}\mathscr{R}_i(\omega_k) + \mathrm{j}\rho^{\mathrm{T}}\mathscr{I}_i(\omega_k), \end{aligned} \tag{5}$$