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Solving linear programming problems under fuzziness and randomness environment using attainment values

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Abstract

In this paper, the author presents a model to measure attainment values of fuzzy numbers/fuzzy stochastic variables. These new measures are then used to convert the fuzzy linear programming problem or the fuzzy stochastic linear programming problem into the corresponding deterministic linear programming problem. Numerical comparisons are provided to illustrate the effectiveness of the proposed method.

Keywords: Fuzzy random variable; Attainment value; Fuzzy linear programming; Fuzzy stochastic linear programming

1. Introduction

Solving fuzzy linear programming problems has received a great deal of attention [10,6,4,2,9,3,8]. Recently, Inuiguchi and Ramik, [7] conducted a comprehensive review of fuzzy optimization. The most popular approach to handle the challenge of solving fuzzy linear programming problems is to convert the fuzzy linear program (FLP) into the corresponding deterministic linear program (LP). An example of this is the approach that compares fuzzy numbers by using some ordering measures of fuzzy numbers, such as the area compensation method [5], the expected mid-point approach [8], the grade of possibility and necessity measures [1], and the signed distance method [3].

In addition, a new line of research in the fuzzy optimization area has been extended to the case of fuzzy stochastic optimization, in which solving fuzzy stochastic linear programming problems is the main concern. Together, stochastic and fuzzy aspects are considered to provide an efficient tool to describe real-life problems where uncertainty and imprecise information are inherent. However, this kind of combination creates a great challenge for researchers to find efficient methods to deal with both fuzzy and stochastic terms. Like the approaches of fuzzy optimization, the general strategy to handle the fuzzy stochastic optimization problem

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is to de-fuzzify and/or de-randomize fuzzy random variables so as to convert the problem into the corresponding deterministic problem. There are two common methods to achieve this. The first is to perform the conversions (de-fuzzify/de-randomize) in a sequential manner [16,17,15]. Although the obtained results of Luhandjula's approaches are LPs, the discretizing process of fuzzy sets via α-levels creates quite a large number of constraints and variables. The second method is to perform both actions at the same time by calculating the expected value of fuzzy random variables [12,13,11,14]. Although the works of Liu proposed a promising measure in terms of expected value of fuzzy random variables, the computation process of these expected values is quite complex and time consuming.

In this paper, the author proposes a new method to solve linear programming problems under the fuzziness and randomness environment. In real-life, fuzzy stochastic linear programming arises in several situations. The parameters of linear programs such as the right-hand-sides (RHSs) and the coefficients of the objective and constraints could all be fuzzy random variables due to the fact that they depend upon many factors. Thus, it is difficult to exactly determine the values of these parameters. Moreover, the factors, which are fluctuating due to an uncertain environment, could cause these parameters to vary. These circumstances often happen in situations wherein the described conditions (objectives, constraints, coefficients) cannot be determined precisely and certainly, such as long-term planning, development strategies [16], engineering design [20], and financial modeling [21].

As an example of fuzzy stochastic linear programming, let's consider the case of a production planning problem where the objective is to minimize the total cost. This objective can be expressed as a fuzzy stochastic variable because the total cost includes the cost of inventory holding, materials, and operation. Production output depends on process parameters (cutting speeds and feed rates, for example) and machine running time. However, machine running time is fluctuating and hard to estimate precisely. In addition, available resources, demand, and constraints' coefficients can also be modeled as fuzzy random variables because of the variances in statistical data due to environmental conditions such as seasonal factors, market prices, and suppliers, which contribute to constraint parameters.

Another example is the case of preventive maintenance. Factory equipment breaks down from time to time, causing losses in production output. Preventive maintenance can be employed to reduce the frequency of such breakdowns, and could involve activities such as inspections, repairs, and replacing components when necessary. These activities are costly in terms of materials, wages, and the loss of production due to downtime while the activities are being conducted. The length of the downtime is also uncertain due to the complexity of the inspection, repair and replacement jobs, as well as the varying skills of maintenance staff. The problem is to determine the frequency of preventative maintenance such that the total downtime, which includes downtime due to breakdowns as well as preventive maintenance, is minimized while ensuring the associated costs do not exceed the available budget. Here, the running time of the machine is also uncertain. Therefore, we would rather consider these times and their associated costs as fuzzy random variables.

Examples such as these motivate the author to propose a new model for solving fuzzy stochastic linear programming problems. In this paper, the attainment measure of fuzzy numbers and/or fuzzy random variables is developed to convert the fuzzy stochastic linear programming problem into the corresponding deterministic LP. The obtained LP, with few additional constraints, is easily solved by standard optimization packages such as LINGO [18] or http://www.lindo.com.

This paper is organized as follows: Firstly, some important results of fuzzy random variables are summarized. Then, in Section 3, the attainment model is developed. Section 4 utilizes the attainment measures to convert the fuzzy linear programming problem into the corresponding deterministic LP. Section 5 presents the solution method for the fuzzy stochastic linear programming problem. Finally, Section 6 presents the conclusions.

2. Fuzzy random variable

In this section, we summarize some important concepts and results of the fuzzy random variable which is the basis for the development of the attainment measures. There are several alternative approaches to define a fuzzy random variable and its characteristics, and for this paper we utilize the results of Luhandjula [17] for the definition.