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ScienceDirect
Fuzzy Sets and Systems 158 (2007) 1961 – 1978
FUZZY sets and systems
www.elsevier.com/locate/fss

Duality results and a dual simplex method for linear programming problems with trapezoidal fuzzy variables

N. Mahdavi-Amiri[a, b, *], S.H. Nasseri[a]

[a] *Department of Mathematical Sciences, Sharif University of Technology, Tehran, Iran*
[b] *Institute for Studies in Theoretical Physics and Mathematics (IPM), Tehran, Iran*

Received 8 October 2005; received in revised form 2 May 2007; accepted 7 May 2007
Available online 25 May 2007

Abstract

Linear programming problems with trapezoidal fuzzy variables (FVLP) have recently attracted some interest. Some methods have been developed for solving these problems by introducing and solving certain auxiliary problems. Here, we apply a linear ranking function to order trapezoidal fuzzy numbers. Then, we establish the dual problem of the linear programming problem with trapezoidal fuzzy variables and hence deduce some duality results. In particular, we prove that the auxiliary problem is indeed the dual of the FVLP problem. Having established the dual problem, the results will then follow as natural extensions of duality results for linear programming problems with crisp data. Finally, using the results, we develop a new dual algorithm for solving the FVLP problem directly, making use of the primal simplex tableau. This algorithm will be useful for sensitivity (or post optimality) analysis when using primal simplex tableaus.

Keywords: Fuzzy variable linear programming; Duality; Ranking function; Dual simplex; Trapezoidal fuzzy number

1. Introduction

Fuzzy set theory has been applied to many disciplines such as control theory and management sciences, mathematical modeling and industrial applications. The concept of fuzzy mathematical programming on general level was first proposed by Tanaka et al. [25] in the framework of the fuzzy decision of Bellman and Zadeh [4]. The first formulation of fuzzy linear programming (FLP) is proposed by Zimmermann [31]. Afterwards, many authors considered various types of the FLP problems and proposed several approaches for solving these problems [5–8,16–20,26]. Chanas [7] showed an application of parametric programming techniques in FLD and obtained the set of solutions maximizing the objective function, being analytically dependent on a parameter. Delgado et al. [8] studied a general model for FLP problems which includes fuzziness both in the coefficients and in the accomplishment of the constraints. Buckley and Feuring [5] considered the extreme case of fully fuzzified linear program with all the parameters and variables as fuzzy numbers. They turned the problem into a multi-objective FLP one. They showed that the fuzzy flexible programming could be used to explore the undominated set to the multi-objective problem and proposed an evolutionary algorithm

* Corresponding author. Tel.: +98 21 66165607.
E-mail addresses: nezamm@sina.sharif.edu, nezamm@sharif.edu (N. Mahdavi-Amiri).

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doi:10.1016/j.fss.2007.05.005

to solve the fuzzy flexible program. Some authors used the concept of comparison of fuzzy numbers for solving FLP problems [6,16]. In effect, most convenient methods are based on the concept of comparison of fuzzy numbers by use of ranking functions [9,17,18,20]. Of course, ranking functions have been proposed by researchers to suit their requirements of the problem under consideration [6,18,20], and conceivably there are no generally accepted criteria for application of ranking functions. Nevertheless, usually in such methods authors define a crisp model which is equivalent to the FLP problem and then use optimal solution of the model as the optimal solution of the FLP problem. A review of some common methods for ranking fuzzy numbers can be seen in [27]. Moreover, a review of the literature concerning fuzzy mathematical programming as well as comparison of fuzzy numbers can be seen in Klir and Yuan [12] and also Lai and Hwang [13]. Some authors considered types of single and multi-objective linear programming problems in which the variables and the right-hand sides of the constraints are fuzzy parameters [10,17–19].

The study of duality theory for fuzzy parameter linear programming problems has attracted researchers in fuzzy decision theory. The duality of fuzzy parameter linear programming was first studied by Rodder and Zimmermann [23]. Verdegay [26] defined the fuzzy dual problem with the help of parametric linear programming and showed that the fuzzy primal and dual problems both have the same fuzzy solution under some suitable conditions. The fuzzy primal and dual linear programming problems with fuzzy coefficients were formulated by using the fuzzy scalar product proposed in [28]. Liu et al. [15] proposed the fuzzy primal and dual problems by considering the fuzzy-max and fuzzy-min in the objective functions as the crisp pattern of linear programming problems. Bector and Chandra [1] discussed duality in FLP based on a modification of the dual formulation stated by Rodder and Zimmermann [23]. Afterwards Bector et al. [2] considered a fuzzy matrix game and proved its equivalence to two crisp linear programming problems which constitute a primal–dual pair in the sense of duality for linear programming with fuzzy parameters (see also [3]). Inuiguchi et al. [11] studied FLP duality in the setting of fuzzy relations. Ramik [22,21] discussed a class of FLP problems based on fuzzy relations and a new concept of duality and deduced the weak and strong duality theorems. Wu [29] offered a concept of fuzzy scalar product and proved the weak and strong duality theorems using a dual fuzzy mathematical programming problem.

The fuzzy variable linear programming (FVLP) problems have been explored by Zimmermann's discussion [32] of the so-called nonsymmetric flexible linear programming (NFLP) problems, where the problem data are considered to be crisp but certain constraints are considered to be "fuzzy inequality" constraints (see also [13,14]). It was shown that under certain conditions the NFLP problem is equivalent to an FVLP problem. Moreover, as we will show later, an FVLP problem is the dual of a fuzzy number linear programming (FNLP) problem, in which the coefficients of the cost function are fuzzy. Therefore, methods for solving FVLP problems can be used for solving both the NFLP and FNLP problems. Maleki et al. [17,18] gave an auxiliary problem, having only fuzzy cost coefficients, for an FVLP problem. In [18], they obtained some results leading to an algorithm for solving the auxiliary problem as a method for solving the FVLP problem (we will see later that their algorithm is a dual algorithm for solving a primal FVLP problem). In [17], they used the algorithm to solve the NFLP problems. Here, we first show that the auxiliary problem given in [18] is indeed the dual of the FVLP problem. This leads us to duality results for the two problems. Our main contributions here are the establishment of duality and complementary slackness, upon the use of certain linear ranking functions to order trapezoidal fuzzy numbers. Using the results, we develop and present a dual simplex algorithm directly using the primal simplex tableau (as opposed to solving the dual problem considered by Maleki et al. [17]). The new algorithm tenders the capability for sensitivity (or post optimality) analysis using primal simplex tableaus. In Section 2, we first give some necessary notations and definitions of fuzzy set theory. Then we provide a discussion of fuzzy numbers and linear ranking functions for ordering them. In particular, a certain linear ranking function for ordering trapezoidal fuzzy numbers is emphasized. The definition of the FVLP problem is given in Section 3. Section 4 explains the notion of fuzzy basic feasible solution. We establish duality for the FVLP problem in Section 5 and deduce the duality results. In Section 6, we develop and present the dual simplex algorithm, using the primal tableau, for solving the FVLP problems. We conclude in Section 7.

2. Preliminaries

2.1. Definitions and notations

We review the fundamental notions of fuzzy set theory, initiated by Bellman and Zadeh [4].