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Necessity measure optimization in linear programming problems with fuzzy polytopes

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Abstract

In this paper, we treat fuzzy linear programming problems with uncertain parameters whose ranges are specified as fuzzy polytopes. The problem is formulated as a necessity measure optimization model. It is shown that the problem can be reduced to a semi-infinite programming problem and solved by a combination of a bisection method and a relaxation procedure. An algorithm in which the bisection method and the relaxation procedure converge simultaneously is proposed. A simple numerical example is given to illustrate the solution procedure.

Keywords: Possibilistic linear programming; Necessity measure; Fuzzy polytope; Semi-infinite programming; Relaxation procedure; Bisection method

1. Introduction

Fuzzy programming approach [6,12,15] is useful and efficient to treat a programming problem under uncertainty. While classical and stochastic programming approach may require a lot of cost to obtain the exact coefficient value or distribution, fuzzy programming approach does not (see [13]). From this fact, fuzzy programming approach will be very advantageous when the coefficients are not known exactly but vaguely specified by human expertise.

Fuzzy programming has been developed under an implicit assumption that all uncertain coefficients are non-interactive with respect to one another. This assumption makes the reduced problem very tractable. The tractability can be seen as one of the advantages of fuzzy programming approaches. However, it is observed that in a simple problem, such as a portfolio selection problem, solutions of models are often intuitively unacceptable because of the implicit assumption (see [9]). This implies that the non-interaction assumption is not sufficient to model the real world problem.

In this sense, we should deal with fuzzy programming problems with interactive uncertain coefficients. However, unfortunately, under a general interaction among uncertain coefficients, the reduced problem may become intractable. This is because we may loose the convexity of the area within which the interactive uncertain coefficients lie (see [10]). Therefore, treatments of interactive uncertain coefficients without loss of tractability of the reduced problem are required in the field of fuzzy programming. Several attempts have been done.

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Rommelfanger and Kresztfalvi [14] proposed to use Yager's parameterized t-norm in order to control the spreads of fuzzy linear function values. The interaction among uncertain parameters is treated indirectly in these approaches. The parameter of Yager's t-norm should be selected for each objective function and for each constraint and the reduced problem is not always a linear programming problem. However, the selection of the parameter of Yager's t-norm will not be very easy.

Inuiguchi and Sakawa [8] treated a fuzzy linear programming with a quadratic membership function. Since a quadratic membership function resembles a multivariate normal distribution, they succeeded to show the equivalence between special models of stochastic linear programming problem and fuzzy linear programming problem. In this approach, though the fractile optimization problem [5] using a necessity measure can be reduced to a convex programming problem, the reduced problem should be solved by an iterative use of quadratic programming techniques.

Inuiguchi and Tanino [10] proposed scenario decomposed fuzzy numbers. In their approach, the interaction between uncertain parameters are expressed by fuzzy if–then rules. They showed that a fuzzy linear programming problem with scenario decomposed fuzzy numbers can be reduced to a linear programming problem with a number of constraints.

Furthermore, Inuiguchi et al. [7] proposed oblique fuzzy vectors. A non-singular matrix shows the interaction among uncertain parameters in an oblique fuzzy vector. It is shown that linear function values of oblique fuzzy vectors can be calculated easily. Owing to this property, fuzzy linear programming problems with oblique fuzzy vectors can be reduced to linear programming problems with a special structure. A solution algorithm utilizing the special structure has been proposed. Oblique fuzzy vectors are able to incorporate knowledge about linear function values of uncertain parameters. However, a non-singular matrix is not always sufficient to express the interaction among uncertain variables in real world problems.

Recently, Inuiguchi and Tanino [11] have introduced a fuzzy polytope to fuzzy linear programming problems. They have shown that the fractile optimization model [5] using a necessity measure can be reduced to a semi-infinite linear programming problem. The reduced problem can be solved by a relaxation procedure. The fuzzy polytope can be constructed from the information about the linear fractional values of uncertain coefficients. Therefore, the fuzzy polytope will be useful when we know the vague values of sums of uncertain coefficients, ratios between two uncertain coefficients or more generally, the linear fractional values of uncertain coefficients.

In this paper, as a continuance of Inuiguchi and Tanino's paper [11] using a fuzzy polytope, we apply a necessity measure optimization model and propose a solution algorithm. In the next section, we describe the problem setting. Then we apply a necessity measure optimization model and show that the problem is reduced to a semi-infinite programming problem but not necessarily to be a linear one in Section 3. In Section 4, we propose a solution algorithm based on a relaxation procedure and a bisection method. In the algorithm, the relaxation procedure and the bisection method converge simultaneously. A simple numerical example is given to illustrate the solution procedure in Section 5. Finally, concluding remarks are given in the last section.

2. Problem statement

In this paper, we treat the following linear programming problem with uncertain parameters:

$$\begin{array}{ll} \text{minimize} & \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}, \\ \text{subject to} & \boldsymbol{a}_i^{\mathrm{T}}\boldsymbol{x} \lesssim_i b_i, \quad i = 1, 2, \ldots, m, \end{array} \tag{1}$$

where $\boldsymbol{c} = (c_1, c_2, \ldots, c_n)^{\mathrm{T}}$, $\boldsymbol{a}_i = (a_{i1}, a_{i2}, \ldots, a_{in})^{\mathrm{T}}$, $i = 1, 2, \ldots, m$ and $\boldsymbol{b} = (b_1, b_2, \ldots, b_m)^{\mathrm{T}}$ are uncertain parameters. Let $\boldsymbol{q}^{\mathrm{T}} = (\boldsymbol{a}_1^{\mathrm{T}}, \boldsymbol{a}_2^{\mathrm{T}}, \ldots, \boldsymbol{a}_m^{\mathrm{T}}, \boldsymbol{b}^{\mathrm{T}}, \boldsymbol{c}^{\mathrm{T}})$. We assume that some linear fractional function values $(\boldsymbol{w}_k^{\mathrm{T}}\boldsymbol{q}+w_{k0})/(\boldsymbol{d}_k^{\mathrm{T}}\boldsymbol{q}+d_{k0})$, $k = 1, 2, \ldots, p$ $(\boldsymbol{d}_k^{\mathrm{T}}\boldsymbol{q} + d_{k0} \neq 0)$ are vaguely known, where $\boldsymbol{w}_k, \boldsymbol{d}_k \in \mathbf{R}^{(mn+m+n)}$, $k = 1, 2, \ldots, p$ are constant vectors, and $w_{k0}, d_{k0} \in \mathbf{R}$, $k = 1, 2, \ldots, p$ are constants. Namely, we assume that the fuzzy boundary of linear fractional function values are known. Based on the linear fractional function value information we may construct an $(mn + m + n)$-dimensional fuzzy set $Q \subseteq \mathbf{R}^{(mn+m+n)}$ with the following membership function:

$$\mu_Q(\boldsymbol{q}) = \min_{k=1,2,\ldots,p} L_k \left(\frac{\frac{\boldsymbol{w}_k^{\mathrm{T}}\boldsymbol{q}+w_{0k}}{\boldsymbol{d}_k^{\mathrm{T}}\boldsymbol{q}+d_{0k}} - \bar{q}_k}{\alpha_k} \right), \tag{2}$$