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Multi-scale reliability analysis and updating of complex systems by use of linear programming

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Abstract

Complex systems are characterized by large numbers of components, cut sets or link sets, or by statistical dependence between the component states. These measures of complexity render the computation of system reliability a challenging task. In this paper, a decomposition approach is described, which, together with a linear programming formulation, allows determination of bounds on the reliability of complex systems with manageable computational effort. The approach also facilitates multi-scale modeling and analysis of a system, whereby varying degrees of detail can be considered in the decomposed system. The paper also describes a method for computing bounds on conditional probabilities by use of linear programming, which can be used to update the system reliability for any given event. Applications to a power network demonstrate the methodology.

Keywords: Complex system; Decomposition; Linear programming; Multi-scale analysis; Power network; Reliability; Systems; Updating

1. Introduction

Critical infrastructures, such as water-, sewage-, gas- and power-distribution systems and highway transportation networks are usually complex systems consisting of numerous structural components. In order to guarantee the reliability of such systems against deterioration or natural and man-made hazards, it is essential to have an efficient and accurate method for estimating the failure probability relative to specified system performance criteria and load hazard. Furthermore, for the purpose of developing emergency or recovery plans, it is often of interest to determine the updated reliability of the system or its components for given scenario events or events that have actually occurred. This paper aims at developing methodologies for such analyses, which are well suited for application to complex systems.

System performance criteria usually are defined either in terms of connectivity between input and output nodes, or in terms of availability of specified levels of "flow" (e.g., water flux or pressure, power voltage) at selected nodes. In the case of a connectivity criterion, each component has only two possible states: connected (functioning) or not connected (failed). In the case of a flow criterion, each component as well as the system can have multiple states, e.g., different levels of flow. Mathematically, the two problems are similar, though a multi-state system usually poses more computational challenges. While the methods developed in this paper are applicable to multi-state systems and brief outlines are given, the main focus of the application is on two-state systems. Applications to multi-state systems are currently under development.

Recently, the authors developed a linear programming (LP) method for computing bounds on the reliability of general, two-state systems in terms of marginal or low-order joint component failure probabilities [1]. For a system with n components, the size of the LP problem to be solved is $N = 2^n$. This number can be enormously large, e.g., for a system with 100 components $N = 1.27 \times 10^{30}$. Obviously, a direct solution of the LP problem for such a system is not possible. To overcome this problem, in this paper, we propose a multi-scale approach, whereby the

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system is decomposed into a number of subsystems and a hierarchy of analyses is performed by considering each subsystem or sets of subsystems separately. In addition to computational advantage, this approach allows consideration of details at the subsystem level, which may not be possible to include in the overall system model.

The LP bounding method is next extended to the computation of conditional probabilities for the purpose of system reliability updating. An iterative solution algorithm with a parameterized LP formulation is proposed for this purpose. Example applications to connectivity problems of an electric power substation and a network demonstrate the methodologies developed in this paper.

2. Review of LP bounds method

We first briefly present the LP formulation for a two-state system and then describe its extension to a multi-state system. Consider a system with n two-state components and let the Boolean variable $s_i \in (0,1)$, $i = 1,\ldots,n$, denote the state of component i with $s_i = 0$ denoting its failure state and $s_i = 1$ denoting its functioning state. There are $N = 2^n$ distinct realizations of the system, each defined by a distinct realization of the vector $\mathbf{s} = [s_1 s_2 \ldots s_n]^\mathrm{T}$. Let p_j, $j = 1,\ldots,N$, denote the probabilities associated with these distinct realizations. These probabilities serve as the unknown variables in the LP formulation. Also let the system function $S = \phi(\mathbf{s})$, $S \in (0,1)$, describe the state of the system with $S = 0$ denoting the failure state and $S = 1$ denoting the functioning state of the system. As described by Barlow and Proschan [2], the system function is a mapping of the component states onto the system states. Its specification requires consideration of the system configuration and performance criterion.

We aim at computing bounds on the system failure probability $P_{\mathrm{sys}} = \Pr(S = 0)$ in terms of marginal and low-order joint component failure probabilities. Denote the marginal component failure probabilities as $P_i = \Pr(s_i = 0)$, $i = 1,\ldots,n$, the bi-component failure probabilities as $P_{ij} = \Pr(s_i = 0 \cap s_j = 0)$ for $i \neq j$, the tri-component failure probabilities as $P_{ijk} = \Pr(s_i = 0 \cap s_j = 0 \cap s_k = 0)$ for $i \neq j \neq k$, etc. Observe that each of the probabilities P_i, P_{ij}, P_{ijk}, etc., is a linear function of the vector $\mathbf{p} = [p_1, p_2, \ldots, p_N]^\mathrm{T}$. For example, P_{ij} is the sum of P_k for all realizations of $\mathbf{s}$ with $s_i = 0$ and $s_j = 0$. Thus, one can write $P_{ij} = \mathbf{a}^\mathrm{T}\mathbf{p}$ with the N-vector $\mathbf{a}$ having elements of 0 or 1. Similar expressions can be written for the marginal or other joint component failure probabilities. Furthermore, the system failure probability is also a linear function of $\mathbf{p}$, as it is the sum of p_k for all realizations of $\mathbf{s}$ for which $\phi(\mathbf{s}) = 0$. Thus, we can write $P_{\mathrm{sys}} = \mathbf{C}^\mathrm{T}\mathbf{p}$, where $\mathbf{C}$ is an N-vector having elements of 0 or 1. Now let $\mathbf{B}_1$ denote the vector of all known marginal or joint component failure probabilities, $\mathbf{B}_2$ denote the vector of all known lower bounds to marginal or joint component failure probabilities, and $\mathbf{B}_3$ denote the vector of all known upper bounds to marginal or joint component failure probabilities. We then formulate the linear program

$$\begin{aligned} \text{minimize/maximize} \quad & P_{\mathrm{sys}} = \mathbf{C}^\mathrm{T}\mathbf{p} \\ \text{subject to} \quad & \mathbf{A}_1\mathbf{p} = \mathbf{B}_1, \\ & \mathbf{A}_2\mathbf{p} \geqslant \mathbf{B}_2, \\ & \mathbf{A}_3\mathbf{p} \leqslant \mathbf{B}_3, \end{aligned} \tag{1a-d}$$

where $\mathbf{A}_1$, $\mathbf{A}_2$, and $\mathbf{A}_3$ are matrices having elements of 0 or 1 and the inequalities in (1c) and (1d) apply to each component of the vector. To the above, we must add the constraints $\sum_{j=1}^{N} p_j = 1$ and $p_j \geqslant 0$, $j = 1,\ldots,N$, to satisfy the rules of probability. These constraints are similar to those in (1b) and (1c). Clearly, minimization (maximization) of the objective function provides the lower (upper) bound to P_{sys}. It is noted that more general equality or inequality expressions in terms of component probabilities, such as $P_i = 3P_j$ or $P_{ij} + P_{jk} \geqslant P_{kl}$, can be easily added to the above formulation as additional linear equality or inequality constraints. The more information (constraint) one provides, the narrower the bounds on P_{sys} become.

As mentioned earlier, the LP formulation can be extended to multi-state systems. Consider a system with n components, the ith component having the m_i states $s_i \in (1, 2, \ldots, m_i)$, $i = 1,2,\ldots,n$. The number of distinct realizations of the system now is $N = \prod_{i=1}^{n} m_i$. Let p_i, $i = 1,2,\ldots,N$, denote the probabilities of these distinct realizations. Suppose these realizations fall into m_{sys} distinct system states so that for each realization the system function $S = \phi(\mathbf{s})$ takes one of the values in the set $S \in (1, 2, \ldots, m_{\mathrm{sys}})$. Using the LP formulation in (1), bounds on the probability that the system is in the ith state, $\Pr(S = i)$, can be obtained in terms of marginal or joint component state probabilities, i.e., the probability that component i is in state k, $P_{i,k} = \Pr(s_i = k)$, the probability that component i is in state k and component j is in state l, $P_{ij,kl} = \Pr(s_i = k \cap s_j = l)$, etc. Note that each of these probabilities is a linear function of $\mathbf{p}$. However, in specifying these probabilities, one must take note of the fact that the states of each component are mutually exclusive and collectively exhaustive events. Thus, constraints such as $\sum_{k=1}^{m_i} P_{i,k} = 1$, $\sum_{k=1}^{m_i}\sum_{l=1}^{m_j} P_{ij,kl} = 1$, etc., must be observed in specifying these probabilities. It is seen that the LP formulation developed in Song and Der Kiureghian [1] for two-state systems is equally applicable to multi-state systems. However, for the same number of components, the LP problem will rapidly grow with increasing number of component states.

As demonstrated in Song and Der Kiureghian [1], the LP formulation has a number of important advantages over other existing methods for computing bounds on the system state probability. These include: (a) any type of information on the marginal and joint component state probabilities can be used, including equality and inequality relations; (b) statistical dependence between component states is easily accounted for in terms of joint component probabilities; (c) the method guarantees the narrowest