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Extensions of TOPSIS for large scale multi-objective non-linear programming problems with block angular structure

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Abstract

This paper focuses on multi-objective large-scale non-linear programming (MOLSNLP) problems with block angular structure. We extend the technique for order preference by similarity ideal solution (TOPSIS) to solve them. Compromise (TOPSIS) control minimizes the measure of distance, provided that the closest solution should have the shortest distance from the positive ideal solution (PIS) as well as the longest distance from the negative ideal solution (NIS). As the measure of "closeness" L_P-metric is used. Thus, we reduce a q-dimensional objective space to a two-dimensional space by a first-order compromise procedure. The concept of a membership function of fuzzy set theory is used to represent the satisfaction level for both criteria. Moreover, we derive a single objective large-scale non-linear programming (LSNLP) problem using the max–min operator for the second-order compromise operation. Finally, a numerical illustrative example is given to clarify the main results developed in this paper.

Keywords: Large scale systems; Multi-objective decision making; Fuzzy set theory; Compromise (satisfactory solution); Positive ideal solution; Negative ideal solution

1. Introduction

Decision-making is the process of selecting a possible course of action from all of the alternatives. In almost all such problems, the multiplicity of criteria for judging the alternative is pervasive. That is, for many such problems, the decision maker wants to attain more than one goal in selecting the course of action while satisfying the constraints dictated by environmental processes and resources [1]. The increasing complexity of modern-day society has brought new problems that involve very large number of variables. Due to the high dimensionality of the problems, it becomes difficult to obtain efficient solutions for them. Most of the large-scale programming problems arising in application have a special structure that can be exploited. One familiar structure is the block angular structure for the constraints that can be used to formulate the sub-problems [2].

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This paper, TOPSIS [3] is extended to solve LSMONLP problems. As it was first developed by Hwang and Yoon [4] for solving a multiple attribute decision making problem. It is based upon the principle that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the longest from the negative ideal solution (NIS). The single criterion of the shortest distance from the given goal or the PIS may not be enough to decision makers. In fact, we might like to have a decision which not only makes as much profit as possible, but also avoids risks as possible. A similar concept has also been pointed out by Zeleny [1].

Recently, Abo-Sinna [5] extended TOPSIS approach to solve multi-objective dynamics programming (MODP) problems. As, he showed that using the fuzzy max–min operator with non-linear membership functions, the obtained solutions that always non-dominated by the original MODP problems.

Deng et al. [6] formulated the inter-company comparison process as a multi-criteria analysis model, and presented an effective approach by modifying TOPSIS for solving such a problem.

Chen [7] extended the concept of TOPSIS to develop a methodology for solving multi-person multi-criteria decision-making problems in a fuzzy environment and he defined the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS).

In the following section, the formulation of LSMONLP problems with block angular structure will be given, for which the Dantzig–Wolfe decomposition method has been successfully applied. The family of d_p-distance and its normalization is discussed in Section 3. The TOPSIS approach is presented in Section 4. Finally, by using TOPSIS, a numerical example in Section 5 is given.

2. Problem formulation

Consider a convex large-scale multi-objective non-linear programming (LSMONLP) problem with the block angular structure forms as follows [2]:

$$\left.\begin{array}{ll}
\min & f_1(x) = f_{11}(x_1) + f_{12}(x_2) + \cdots + f_{1N}(x_N) \\
\min & f_2(x) = f_{21}(x_1) + f_{22}(x_2) + \cdots + f_{2N}(x_N) \\
 & \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
\min & f_Q(x) = f_{Q1}(x_1) + f_{Q2}(x_2) + \cdots + f_{QN}(x_N) \\
\text{s.t.} & g_1(x) = g_{11}(x_1) + g_{12}(x_2) + \cdots + g_{1N}(x_N) \leqslant 0 \\
 & g_2(x) = g_{21}(x_1) + g_{22}(x_2) + \cdots + g_{2N}(x_N) \leqslant 0 \\
 & \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
 & g_{m_0}(x) = g_{m_01}(x_1) + g_{m_02}(x_2) + \cdots + g_{m_0N}(x_N) \leqslant 0 \\
 & \qquad h_1(x_1) \qquad\qquad\qquad\qquad \leqslant 0 \\
 & \qquad\qquad\qquad h_2(x_2) \qquad\qquad \leqslant 0 \\
 & \qquad\qquad\qquad\qquad \ddots \\
 & \qquad\qquad\qquad\qquad\qquad h_N(x_N) \leqslant 0
\end{array}\right\} \tag{1}$$

where $f_q(x)$, $q = 1, \ldots, Q$ are a distinct functions of the decision vector $x = (x_1, x_2, \ldots, x_N) \in R^n, x_j \in R^{n_j}, j = 1, \ldots, N$, are n_j dimensional vectors of decision variables, $g_l(x) \leqslant 0, l = 1, \ldots, m_0$ are m_0 coupling constraints, $h_j(x_j)$, $j = 1, \ldots, N$, are r_j dimensional constraints with respect to x_j, and $f_{qj}(x_j), g_{lj}(x_j)$, $q = 1, \ldots, Q, l = 1, \ldots, m_0$, $j = 1, \ldots, N$, are non-linear functions with respect to x_j [2].

The kth subproblem [Sub(S_1)] can be defined as

For each $k = 1, \ldots, N$,

Sub (S_1):