Optimality conditions for linear programming problems with fuzzy coefficients

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Received 22 November 2006; accepted 28 September 2007

Abstract

The optimality conditions for linear programming problems with fuzzy coefficients are derived in this paper. Two solution concepts are proposed by considering the orderings on the set of all fuzzy numbers. The solution concepts proposed in this paper will follow from the similar solution concept, called the nondominated solution, in the multiobjective programming problem. Under these settings, the optimality conditions will be naturally elicited.

Keywords: Fuzzy numbers; Nondominated solutions; (crisp) Fuzzy constraints

1. Introduction

The occurrence of fuzziness in the real world is inevitable owing to some unexpected situations. Therefore, imposing fuzziness upon conventional optimization problems becomes an interesting research topic. The collection of papers on fuzzy optimization edited by Słowiński [1] and Delgado et al. [2] gives the main stream of this topic. Lai and Hwang [3,4] also give an insightful survey. On the other hand, the book edited by Słowiński and Teghem [5] provides comparisons between fuzzy optimization and stochastic optimization for multiobjective programming problems.


The duality of the fuzzy linear programming problem was firstly studied by Rodder and Zimmermann [20] considering the economic interpretation of the dual variables. After that, many interesting results regarding the duality of the fuzzy linear programming problem was investigated by Bector et al. [21–23], Liu et al. [24], Ramík [25], Verdegay [26] and Wu [27]. In this paper, we investigate the optimality conditions for linear programming problems with fuzzy coefficients.
Let $\mathbb{R}$ be the set of all real numbers. The fuzzy subset $\tilde{a}$ of $\mathbb{R}$ is defined by a function $\xi_{\tilde{a}} : \mathbb{R} \rightarrow [0, 1]$, which is called a membership function. The $\alpha$-level set of $\tilde{a}$, denoted by $\tilde{a}_\alpha$, is defined by $\tilde{a}_\alpha = \{x \in \mathbb{R} : \xi_{\tilde{a}}(x) \geq \alpha\}$ for all $\alpha \in (0, 1]$. The 0-level set $\tilde{a}_0$ is defined as the closure of the set $\{x \in \mathbb{R} : \xi_{\tilde{a}}(x) > 0\}$, i.e., $\tilde{a}_0 = \operatorname{cl}(\{x \in \mathbb{R} : \xi_{\tilde{a}}(x) > 0\})$.

**Definition 2.1.** We denote by $\mathcal{F}(\mathbb{R})$ the set of all fuzzy subsets $\tilde{a}$ of $\mathbb{R}$ with membership function $\xi_{\tilde{a}}$ satisfying the following conditions:

1. $\tilde{a}$ is normal, i.e., there exists an $x \in \mathbb{R}$ such that $\xi_{\tilde{a}}(x) = 1$;
2. $\xi_{\tilde{a}}$ is quasi-concave, i.e., $\xi_{\tilde{a}}(\lambda x + (1 - \lambda)y) \geq \min\{\xi_{\tilde{a}}(x), \xi_{\tilde{a}}(y)\}$ for all $x, y \in \mathbb{R}$ and $\lambda \in [0, 1]$;
3. $\xi_{\tilde{a}}$ is upper semicontinuous, i.e., $\{x \in \mathbb{R} : \xi_{\tilde{a}}(x) \geq \alpha\} = \tilde{a}_\alpha$ is a closed subset of $U$ for each $\alpha \in (0, 1]$;
4. the 0-level set $\tilde{a}_0$ is a compact subset of $\mathbb{R}$.

The member $\tilde{a}$ in $\mathcal{F}(\mathbb{R})$ is called a fuzzy number.

Suppose now that $\tilde{a} \in \mathcal{F}(\mathbb{R})$. From Zadeh [28], the $\alpha$-level set $\tilde{a}_\alpha$ of $\tilde{a}$ is a convex subset of $\mathbb{R}$ for each $\alpha \in [0, 1]$ from condition (ii). Combining this fact with conditions (iii) and (iv), the $\alpha$-level set $\tilde{a}_\alpha$ of $\tilde{a}$ is a compact and convex subset of $\mathbb{R}$ for each $\alpha \in [0, 1]$, i.e., $\tilde{a}_\alpha$ is a closed interval in $\mathbb{R}$ for each $\alpha \in [0, 1]$. Therefore, we also write $\tilde{a}_\alpha = [\tilde{a}_\alpha^L, \tilde{a}_\alpha^U]$.

**Definition 2.2.** Let $\tilde{a}$ be a fuzzy number. We say that $\tilde{a}$ is nonnegative if $\tilde{a}_\alpha^L \geq 0$ for all $\alpha \in [0, 1]$. We say that $\tilde{a}$ is positive if $\tilde{a}_\alpha^L > 0$ for all $\alpha \in [0, 1]$. We say that $\tilde{a}$ is nonpositive if $\tilde{a}_\alpha^U \leq 0$ for all $\alpha \in [0, 1]$. We say that $\tilde{a}$ is negative if $\tilde{a}_\alpha^U < 0$ for all $\alpha \in [0, 1]$.

**Remark 2.1.** Let $\tilde{a}$ be a fuzzy number. Then $\tilde{a}_\alpha^L \leq \tilde{a}_\alpha^U$ for all $\alpha \in [0, 1]$. Therefore if $\tilde{a}$ is nonnegative then $\tilde{a}_\alpha^L \geq 0$ and $\tilde{a}_\alpha^U \geq 0$ for all $\alpha \in [0, 1]$, and if $\tilde{a}$ is positive then $\tilde{a}_\alpha^L > 0$ and $\tilde{a}_\alpha^U > 0$ for all $\alpha \in [0, 1]$. We can have similar consequences for nonpositive and negative fuzzy numbers.

Let “$\odot$” be any binary operations $\oplus$ or $\otimes$ between two fuzzy numbers $\tilde{a}$ and $\tilde{b}$. The membership function of $\tilde{a} \odot \tilde{b}$ is defined by

$$
\xi_{\tilde{a} \odot \tilde{b}}(z) = \sup_{x,y,z} \min\{\xi_{\tilde{a}}(x), \xi_{\tilde{b}}(y)\}
$$

using the extension principle in Zadeh [29], where the operations $\odot = \oplus$ and $\otimes$ correspond to the operations $\circ = +$ and $\times$, respectively. Then we have the following results.

**Proposition 2.1.** Let $\tilde{a}, \tilde{b} \in \mathcal{F}(\mathbb{R})$. Then we have

(i) $\tilde{a} \oplus \tilde{b} \in \mathcal{F}(\mathbb{R})$ and

$$
(\tilde{a} \oplus \tilde{b})_\alpha = \left[\tilde{a}_\alpha^L + \tilde{b}_\alpha^L, \tilde{a}_\alpha^U + \tilde{b}_\alpha^U\right];
$$

(ii) $\tilde{a} \otimes \tilde{b} \in \mathcal{F}(\mathbb{R})$ and

$$
(\tilde{a} \otimes \tilde{b})_\alpha = \left[\min\{\tilde{a}_\alpha^L \tilde{b}_\alpha^L, \tilde{a}_\alpha^L \tilde{b}_\alpha^U, \tilde{a}_\alpha^U \tilde{b}_\alpha^L, \tilde{a}_\alpha^U \tilde{b}_\alpha^U\}, \max\{\tilde{a}_\alpha^L \tilde{b}_\alpha^L, \tilde{a}_\alpha^L \tilde{b}_\alpha^U, \tilde{a}_\alpha^U \tilde{b}_\alpha^L, \tilde{a}_\alpha^U \tilde{b}_\alpha^U\}\right].
$$

The following proposition is very useful for discussing the optimality conditions.

**Proposition 2.2.** Let $\tilde{a}$ be a nonnegative fuzzy number and $\tilde{b}$ be a nonpositive fuzzy number. If $\tilde{a} \otimes \tilde{b} = 0$, then $\tilde{a}_\alpha^L \tilde{b}_\alpha^L = \tilde{a}_\alpha^L \tilde{b}_\alpha^U = \tilde{a}_\alpha^U \tilde{b}_\alpha^L = \tilde{a}_\alpha^U \tilde{b}_\alpha^U = 0$ for all $\alpha \in [0, 1]$. 

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