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A class of multiobjective linear programming models with random rough coefficients

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Abstract

In the present paper, we concentrate on dealing with a class of multiobjective programming problems with random rough coefficients. We first discuss how to turn a constrained model with random rough variables into crisp equivalent models. Then an interactive algorithm which is similar to the interactive fuzzy satisfying method is introduced to obtain the decision maker's satisfying solution. In addition, the technique of random rough simulation is applied to deal with general random rough objective functions and random rough constraints which are usually hard to convert into their crisp equivalents. Furthermore, combined with the techniques of random rough simulation, a genetic algorithm using the compromise approach is designed for solving a random rough multiobjective programming problem. Finally, illustrative examples are given in order to show the application of the proposed models and algorithms.

Keywords: Rough variable; Random rough variable; Trust measure; Chance measure; Random rough simulation; Genetic algorithm; Compromise solution

1. Introduction

In realistic decision making situations, there are cases in which a decision must be made on the basis of uncertain data. For dealing with such decision making problems including uncertainty, many scholars have introduced models including random or fuzzy or rough variables to formulate the uncertainty, such as Moran [1], Liu [2], Amelia Bilbao Terol [3], Hsien-Chung Wu [4] and Slowinski [5]. They all constructed models for solving the practical and complex problems according to the following mathematical programming problem:

$$\max f(x,\xi)$$
$$\text{s.t.} \begin{cases} g_i(x,\xi) \leq 0, \quad i = 1, 2, \ldots, p \\ x \in X, \end{cases}$$

where the set $X \subset \mathcal{R}^N$, $f(x,\xi)$ is the objective function, $g_i(x,\xi)$ $(i = 1, 2, \ldots, p)$ are constraints, and ξ is the uncertain variable. In [1], Moran introduced a SALSA (Stochastic Approach for Link-Structure Analysis) algorithm

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to examine random walks on graphs derived from the link structure. In [2], Liu presented a stochastic expected model, stochastic chance constrained programming and stochastic dependent chance programming. In [3], Amelia Bilbao Terol designed flexible decision making models in the distance metric optimization framework for problems including parameters which are represented by fuzzy numbers. In [4], Hsien-Chung Wu made the fuzzy numbers embed into a normed space, then invoked the scalarization techniques to evaluate the multiobjective programming problems with fuzzy coefficients. In [5], Slowinski applied the method of rough sets to solve an uncertain problem in the medical domain.

In these models, randomness, fuzziness and roughness are considered as separate aspects. However, in a decision making process, we may face a hybrid uncertain environment. Jun Li, Jiuping Xu and Mitsuo Gen [6] discussed a class of multiobjective programming problems with fuzzy random coefficients. Tadeusz Gerstenkorn and Jacek Manko [7] introduced the notion of bifuzzy probabilistic sets and discussed some properties of these sets. Peng and Liu [8] introduced a novel concept of a birandom variable and exhibited the framework of birandom variables. However, there is a class of uncertain problems with randomness and roughness simultaneously which are still paid less attention. For example, in a supply–demand problem, the demand accords with normal distribution but the expected value is a rough variable, i.e., the minimum mean demand amount varies in an interval and the max mean demand amount also varies in an interval. In [2,9], Liu introduced the concept of random rough variable and presented a random rough expected value model and chance constrained programming:

$$\begin{aligned}&\max\{f_1, f_2, \ldots, f_m\}\\&\text{s.t.}\begin{cases}\mathrm{Ch}\{f_i(x,\xi) \geq f_i\}(\gamma_i) \geq \delta_i, & i = 1, 2, \ldots, m\\ \mathrm{Ch}\{g_r(x,\xi) \leq 0\}(\eta_r) \geq \theta_r, & r = 1, 2, \ldots, p\\ x \in X,\end{cases}\end{aligned}$$

where ξ is a random rough variable, $\gamma_i, \delta_i, \eta_r$ and θ_r are predetermined confidence levels, $i = 1, 2, \ldots, m, r = 1, 2, \ldots, p$. This is a useful tool for dealing with uncertain problems with randomness and roughness simultaneously. In this paper, on the basis of the chance measure of random rough variable [9], the tr–pr constrained multiobjective programming model can be easily given. Then we discuss the consistency with the models when random rough variable $\bar{\tilde{\xi}}$ degenerates to a rough variable $\bar{\xi}$ or a random variable $\tilde{\xi}$. Thereby, it is proved that the proposed models are reasonable. However, it is difficult to get the optimal solution of many problems with random rough coefficients. Thus, we propose a crisp equivalent model aimed at this kind of multiobjective problem. Then we apply the technique of random rough simulation to deal with general random rough objective functions and random rough constraints. This is an efficient method and the convergence of the random rough simulation can be guaranteed [9]. Finally, combined with the techniques of random rough simulation, a genetic algorithm using the compromise approach is designed for solving a random rough multiobjective programming problem.

The rest of this paper is organized as follows. In Section 2 we recall some definitions and properties for random variables, rough variables and random rough variables. Then the tr–pr constrained multiobjective programming model is introduced in Section 3. A crisp equivalent model is proposed for a special type of random rough variable, and an interactive random satisfying method is adopted to obtain the decision maker's satisfactory solution. In Sections 4 and 5, we respectively represent random rough simulation and a random rough simulation-based genetic algorithm using a compromise approach. In Section 6, two illustrative examples are given in order to show the application of the proposed models and algorithms. Finally, the conclusions are given in Section 7.

2. Preliminaries

People have used algebraic, topological, logical and constructive methods to study rough sets and try to formulate axiomatic systems for classical rough sets from different viewpoints [5,10–12]. In this paper, we assume the basic definition and properties of rough variables, and refer the reader to [2]. The following will be used extensively.

Definition 2.1. Let Λ be a nonempty set, $\mathcal{A}$ be a σ-algebra of subsets of Λ, Δ be an element in $\mathcal{A}$, and π be a nonnegative, real-valued, additive set function. Then $(\Lambda, \Delta, \mathcal{A}, \pi)$ is called a rough space.

Definition 2.2. A rough variable ξ on the rough space $(\Lambda, \Delta, \mathcal{A}, \pi)$ is a function from Λ to the real line $\Re$ such that for every Borel set O of $\Re$, we have