

Construction of aggregation functions from data using linear programming

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Abstract

This article examines the construction of aggregation functions from data by minimizing the least absolute deviation criterion. We formulate various instances of such problems as linear programming problems. We consider the cases in which the data are provided as intervals, and the outputs ordering needs to be preserved, and show that linear programming formulation is valid for such cases. This feature is very valuable in practice, since the standard simplex method can be used.

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1. Introduction

Aggregation functions play an important role in several areas, including fuzzy logic, decision making, expert systems, risk analysis and image processing. Recent books [15,19,21,31,48] provide a comprehensive overview of aggregation functions and methods of their construction. The purpose of aggregation functions is to combine several input values into a single output value, which in some sense represents all the inputs. Typically the inputs and outputs are real numbers, often from $[0, 1]$, although other choices are possible, e.g., discrete sets, intervals and linguistic labels. Notable examples are weighted means, medians, ordered weighted averaging (OWA) functions, discrete Choquet and Sugeno integrals, triangular norms and conorms, uninorms and nullnorms.

Often aggregation functions need to be constructed based on specified mathematical properties and some data. Consider the following problem setting. A decision maker wants to automate the decision process and to construct an aggregation function for a given number of inputs. She needs to ensure that (a) the aggregation function satisfies a number of properties (e.g., idempotency, symmetry, possessing a neutral element, etc.) and (b) that for certain prototypical inputs its outputs are consistent with the desired values. These values can come from a mental experiment: What should be the aggregated value of the input like $x = (1, 1, 0, \frac{1}{2})$? They can also come from a controlled experiment, like questioning the experts in a field. One such experiment was reported in [16] in the context of a medical decision support system. Several medical specialists were presented with a number of prototypical cases and were asked to express the strength of their opinion on a numerical scale. In case of disagreement they were asked to reach consensus, which often resulted

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in an interval of equally possible outputs. These input–output pairs were used to construct a suitable aggregation function.

There are different approaches to identification of weights in multicriteria decision making, typically in the context of additive or multiplicative utility functions. Saaty’s analytic hierarchy process (AHP) [41] (and its derivatives) is the most common approach. A recent overview of various methods in this area is in [28], see also [36,49] for a sample of recent approaches based on AHP. Some alternative approaches include SMART [27], the centroid method [45] and testing the form of the utility function [42], see a recent comparative overview [43].

There is also a number of alternative methods for fitting aggregation functions, other than weighted arithmetic means, to empirical data [15,48,6]. They range from identifying weighted quasi-arithmetic means and OWA functions [12,46,47,29,40] to fitting additive generators of triangular norms, conorms and uninorms [5,7], from constructing fuzzy measures [14,37,32,30,2] to purely interpolative type functions [9,13]. Such aggregation functions are frequently encountered in fuzzy control (e.g., triangular norms and conorms) and expert systems (the classical MYCIN’s and PROSPECTOR’s aggregation functions are uninorms [23]). In most of the mentioned studies, the least squares criterion was optimized, subject to constraints on the variable parameters, which ensure consistency with a priori specified properties. This typically translates into a quadratic programming problem, for which many numerically efficient algorithms are available. In some cases multistart local search and genetic algorithms were used to solve more general nonlinear optimization problems, but this comes at the expense of greatly increased computational cost and often suboptimal solutions.

In this article we concentrate on the least absolute deviation (LAD) criterion [17], which allows one to translate the fitting problem into a linear programming (LP) problem. LAD criterion is often used in regression because (a) it is less sensitive to outliers and (b) LP formulation allows one to deal with a much greater number of parameters. Both features are important for constructing aggregation functions. In addition to that, we show that the regression problem can be easily extended to cases where the data are given as intervals and where preservation of the outputs ordering is important, and this can be done without sacrificing the LP formulation. This is of course very desirable, since LP problems are easily solved numerically even for large numbers of parameters and constraints.

The article is structured as follows. Section 2 provides several definitions, formulates the LAD regression problem and provides details of its LP formulation. Section 3 analyzes the LAD regression problem for several popular families of aggregation functions. It includes estimation of weights of weighted arithmetic means and OWA functions as special cases; however, the range of applicability of the discussed methods is much broader than that of other methods based on additivity assumption (like the AHP, SMART and the centroid methods), and includes fitting the generating functions of quasi-arithmetic means, triangular norms, conorms, uninorms, and T–S functions, as well as fitting coefficients of fuzzy measures. In Section 4 we consider an important case where either the outputs or both the inputs and the outputs are specified as intervals, and provide its LP formulation. The paper ends with the concluding remarks.

2. Preliminaries

Recent comprehensive overviews of aggregation functions are given in [15,19,21,31,48], from which we took some relevant definitions, see also [25,26].

Definition 1 (*Aggregation function*). An aggregation function is a function of $n > 1$ arguments that maps the (n -dimensional) unit cube onto the unit interval $f : [0, 1]^n \rightarrow [0, 1]$, with the properties

- (i) $f(\underbrace{0, 0, \dots, 0}_{n\text{-times}}) = 0$ and $f(\underbrace{1, 1, \dots, 1}_{n\text{-times}}) = 1$.
- (ii) $x \leq y$ implies $f(x) \leq f(y)$ for all $x, y \in [0, 1]^n$.

The vector inequality is understood componentwise. Aggregation functions may possess various properties, which often classify them into special classes.

- An aggregation function f is called averaging if it is bounded (for all $x \in [0, 1]^n$) by

$$\min(x) = \min_{i=1, \dots, n} x_i \leq f(x) \leq \max_{i=1, \dots, n} x_i = \max(x).$$

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