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Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution

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ABSTRACT

This paper discusses full fuzzy linear programming (FFLP) problems of which all parameters and variable are triangular fuzzy numbers. We use the concept of the symmetric triangular fuzzy number and introduce an approach to defuzzify a general fuzzy quantity. For such a problem, first, the fuzzy triangular number is approximated to its nearest symmetric triangular number, with the assumption that all decision variables are symmetric triangular. An optimal solution to the above-mentioned problem is a symmetric fuzzy solution. Every FLP models turned into two crisp complex linear problems; first a problem is designed in which the center objective value will be calculated and since the center of a fuzzy number is preferred to (its) margin. With a special ranking on fuzzy numbers, the FFLP transform to multi objective linear programming (MOLP) where all variables and parameters are crisp.

1. Introduction

Concept of decision analysis in fuzzy environment was first proposed by Bellman and Zadeh [1]. Some researchers have proposed several fuzzy models [2–7]. Other kinds of FLPs have also been considered in [2,8–18]. However, in all of the above-mentioned works, those cases of FLP have been studied in which not all parts of the problem were assumed to be fuzzy, e.g., only the right hand side or the objective function coefficients were fuzzy; or the variables were not fuzzy. In this paper, we consider a problem in which that all variables and parameters are fuzzy triangular asymmetric numbers with certain conditions. Fully fuzzified linear programming problem, solution and duality have been studied in [19]. The authors in [19] used the possibilities mean value and variance of the fuzzy numbers and considered symmetric triangular fuzzy numbers data. In this manner, the coefficient vector in the objective function or the coefficient matrix of the constraints contain fuzzy elements. We will propose the nearest symmetric triangular approximate (defuzzification approach). Defuzzification methods have been widely studied for some years and were applied to fuzzy control and fuzzy expert systems. The major idea behind these methods is to obtain a typical value from a given fuzzy set according to some specified characters (center, fuzziness, gravity, median, etc.). In this paper, we use the concept of the symmetric triangular fuzzy number and introduce an approach to defuzzify a general fuzzy quantity. The basic idea of the new method is to obtain the "nearest" symmetric triangular approximation of fuzzy numbers which is a fuzzy quantity defined in [20]. Fuzzy linear programming with a multiple objective linear programming problem (MOLPP) has been considered in [17,21,11]. For solving a full fuzzy linear programming problem, we consider the ranking of the constraints. The paper is organized as follows: In Section 2, we note symmetric triangular fuzzy numbers, then we have a multiple objective linear programming problem. This MOLPP has two objective

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functions whit ordinal preference. Then we use the lexicographic method to solve it, and because of the existence of fuzzy inequalities in properties of fuzzy numbers and the MOLPP, in Section 3 we explain a fuzzy linear programming; in Section 4 we have an example; conclusion is drawn in Section 5.

2. Preliminaries

We represent an arbitrary fuzzy number by an ordered pair of functions $\tilde{u} =: (\underline{u}(r), \overline{u}(r))$, $0 \leqslant r \leqslant 1$, which satisfy the following requirements:

- $\underline{u}(r)$ is a bounded left continuous nondecreasing function over [0,1].
- $\overline{u}(r)$ is a bounded left continuous nonincreasing function over [0,1].
- $\underline{u}(r)$ and $\overline{u}(r)$ are right continuous at 0.
- $\underline{u}(r) \leqslant \overline{u}(r)$, $0 \leqslant r \leqslant 1$.

A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha$, $0 \leqslant r \leqslant 1$.

Definition 2.1. $C_{\tilde{u}} = Core(\tilde{u}) = \overline{u}(1) = \underline{u}(1)$; and $w^L_{\tilde{u}} = C_{\tilde{u}} - \underline{u}(0) \geqslant 0$ and $w^R_{\tilde{u}} = \overline{u}(0) - C_{\tilde{u}} \geqslant 0$ are the left and right margins of ũ.

Definition 2.2. The fuzzy number $\tilde{t} =: (C_{\tilde{t}} - w^L_{\tilde{t}} + w^L_{\tilde{t}}r, C_{\tilde{t}} + w^R_{\tilde{t}} - w^R_{\tilde{t}}r) =: (C_{\tilde{t}}, w^L_{\tilde{t}}, w^R_{\tilde{t}}), 0 \leqslant r \leqslant 1$ is an asymmetric triangular fuzzy number $\widehat{ATFN}$. As a matter of fact $C_{\tilde{t}} - w^L_{\tilde{t}} + w^L_{\tilde{t}}r = \underline{t}(r)$ and $C_{\tilde{t}} + w^R_{\tilde{t}} - w^R_{\tilde{t}}r = \overline{t}(r)$ where $C_{\tilde{t}}, w^L_{\tilde{t}}, w^R_{\tilde{t}} \in \Re$. Let $\widehat{A.S.T}$ be the set of all $\widehat{ATFN}$.

A conventional fuzzy number is the symmetric triangular fuzzy number $S[x_o,\sigma]$ where $w^L_S = w^R_S = \sigma$ centered at x_o with basis 2σ. Its parametric form is $S[x_o,\sigma] = :(x_o - \sigma + r(\sigma), x_o + \sigma - r(\sigma)) := (x_o;\sigma), 0 \leqslant r \leqslant 1$ which $x_o, \sigma \in \Re$, x_o is the center and $\sigma \geqslant 0$ is the margin of $S[x_o,\sigma]$ and it is called symmetric triangular fuzzy number ($\widehat{STFN}$).

Let $\widehat{S.T}$ be the set of all $\widehat{STFN}$.

Therefore the following properties for the $\widehat{A.S.T}$ are satisfying in the $\widehat{S.T}$.

Definition 2.3. Let $\tilde{t} = (C_1, w^L_1, w^R_1), \tilde{u} = (C_2, w^L_2, w^R_2) \in \widehat{A.S.T}$ and $k \in \Re$, by using extension principal we can define:

1. $\tilde{t} = \tilde{u}$ if and only if $C_1 = C_2$; and $w^L_1 = w^L_2$ and $w^R_1 = w^R_2$.
2. $\tilde{t} + \tilde{u} = (C_1 + C_2, w^L_1 + w^L_2, w^R_1 + w^R_2)$.
3.

$$k\tilde{t} = \begin{cases} (kC_1, kw^L_1, kw^R_1), & k \geqslant 0 \\ (kC_1, -kw^R_1, -kw^L_1), & k < 0 \end{cases} \tag{2.1}$$

Definition 2.4. For two fuzzy numbers in parametric forms $\tilde{t} = (\underline{t}(r), \overline{t}(r))$, $\tilde{u} = (\underline{u}(r), \overline{u}(r))$ we have: $\tilde{t}\tilde{u} = \tilde{h} = (\underline{h}(r), \overline{h}(r))$ where $\underline{h}(r) = Min\{\underline{t}(r)\underline{u}(r), \overline{t}(r)\overline{u}(r), \overline{t}(r)\underline{u}(r), \underline{t}(r)\overline{u}(r)\}, \overline{h}(r)) = Max\{\underline{t}(r)\underline{u}(r), \overline{t}(r)\overline{u}(r), \overline{t}(r)\underline{u}(r), \underline{t}(r)\overline{u}(r)\}$ for example for two positive $\widehat{A.S.T}s$ $\tilde{t} = (C_{\tilde{t}} + w^L_{\tilde{t}}(r-1), C_{\tilde{t}} + w^R_{\tilde{t}}(1-r))$, and $\tilde{u} = (C_{\tilde{u}} + w^L_{\tilde{u}}(r-1), C_{\tilde{u}} + w^R_{\tilde{u}}(1-r))$ where $C_{\tilde{t}} - w^L_{\tilde{t}} \geqslant 0$ and $C_{\tilde{u}} - w^L_{\tilde{u}} \geqslant 0$ we have: $\tilde{t}\tilde{u} = (C_{\tilde{t}}C_{\tilde{u}} + C_{\tilde{t}}w^L_{\tilde{u}}(r-1) + w^L_{\tilde{t}}(r-1)C_{\tilde{u}} + w^L_{\tilde{t}}w^L_{\tilde{u}}(r-1)^2, C_{\tilde{t}}C_{\tilde{u}} + C_{\tilde{t}}w^R_{\tilde{u}}(1-r) + w^R_{\tilde{t}}(1-r)C_{\tilde{u}} + w^R_{\tilde{t}}w^R_{\tilde{u}}(1-r)^2)$. Suppose $\widetilde{A} \in E^{n\times n}$, E is the eucludian space of fuzzy numbers. $\widetilde{X} = (\tilde{t}_1, \tilde{t}_2, \ldots, \tilde{t}_n)^T$ and $\widetilde{Y} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n)^T$ are $\widehat{ATFN}$ vectors this means that $\widetilde{X}, \widetilde{Y} \in \widehat{A.S.T}^n$. Now we have

1. $Core(\widetilde{X} + \widetilde{Y}) = Core(\widetilde{X}) + Core(\widetilde{Y})$.
2. $Core(\widetilde{A}\widetilde{X}) = Core(\widetilde{A})Core(\widetilde{X})$.
3. $\widetilde{A}(\widetilde{X} + \widetilde{Y}) = \widetilde{A}\widetilde{X} + \widetilde{A}\widetilde{Y}$.

Definition 2.5 (*Ordering on* $\widehat{S.T}$). Let $\tilde{t} = (x_{o_1}; \sigma_1)$ and $\tilde{u} = (x_{o_2}; \sigma_2)$ are $\widehat{STFN}$s. We say $\tilde{t} <^* \tilde{u}$ if and only if:

1. $x_{o_1} < x_{o_2}$ or
2. $x_{o_1} = x_{o_2}$ and $\sigma_1 > \sigma_2$.

In the case equality we have $\tilde{t} =^* \tilde{u}$ if and only if $((x_{o_1} = x_{o_2}) \wedge (\sigma_1 = \sigma_2))$.

And $\tilde{t} \leqslant^* \tilde{u}$ if and only if $(\tilde{t} <^* \tilde{u} \vee \tilde{t} =^* \tilde{u})$ it means that:

$(x_{o_1} < x_{o_2}) \vee [(x_{o_1} = x_{o_2} \wedge \sigma_1 > \sigma_2) \vee (x_{o_1} = x_{o_2} \wedge \sigma_1 = \sigma_2)]$, that is equivalent with the following relation: $(x_{o_1} < x_{o_2}) \vee [(x_{o_1} = x_{o_2} \wedge \sigma_1 \geqslant \sigma_2)]$.

Its clear that by this definition $\widehat{STFN}$s have the triple axiom. For any $\tilde{t}, \tilde{u} \in \widehat{S.T}$ we have only one of these $(\tilde{t} <^* \tilde{u}, \tilde{t} = \tilde{u}, \tilde{t} >^* \tilde{u})$.