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Application of particle swarm optimization algorithm for solving bi-level linear programming problem

R.J. Kuo [a,*], C.C. Huang [b]

[a] Department of Industrial Management, National Taiwan University of Science and Technology, No. 43, Section 4, Kee-Lung Road, Taipei, 106, Taiwan, ROC

[b] Department of Industrial Engineering and Management, National Taipei University of Technology, No. 1, Section 3, Chung-Hsiao East Road, Taipei, 106, Taiwan, ROC

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ABSTRACT

Bi-level linear programming is a technique for modeling decentralized decision. It consists of the upper-level and lower-level objectives. This paper attempts to develop an efficient method based on particle swarm optimization (PSO) algorithm with swarm intelligence. The performance of the proposed method is ascertained by comparing the results with genetic algorithm (GA) using four problems in the literature and an example of supply chain model. The results illustrate that the PSO algorithm outperforms GA in accuracy.

1. Introduction

Multi-level programming techniques are developed to solve decentralized planning problems with multiple decision makers in a hierarchical organization. The bi-level linear programming problem (BLPP) is a special case of multi-level linear programming problems with a two-level structure [1,2]. Most of the mathematical programming models deal with a single decision maker and a single objective function and are used for centralized planning systems. The BLPP on the other hand is developed for decentralized planning systems in which the upper level is termed as the leader and the lower level pertains to the objective of the follower. In the BLPPs, each decision maker tries to optimize its own objective function without considering the objective of the other party, but the decision of each party affects the objective value of the other party as well as the decision space.

There already have been some methods for solving BLPPs, like methods based on vertex enumeration and meta-heuristics. In this study, an attempt is made to employ particle swarm optimization (PSO) algorithm for solving BLPPs due to its promising performance in optimization problems. Four problems taken from the literature are adopted to test the proposed algorithm's performance. The experimental results indicate that PSO algorithm outperforms genetic algorithm (GA) in accuracy and has better stability. In addition, an example of supply chain model also reveals that PSO algorithm is a suitable approach for solving BLPPs.

The rest of this paper is organized as follows. Section 2 provides basic concept of BLPPs and PSO algorithm. The proposed PSO algorithm for solving BLPPs will be presented in Section 3, while Section 4 makes a thorough discussion on computational experiences. Finally, the concluding remarks are made in Section 5.

2. Background

This section will briefly present the background for bi-level programming and PSO algorithm.

* Corresponding author.
E-mail address: rjkuo@mail.ntust.edu.tw (R.J. Kuo).

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2.1. Bi-level linear programming problem (BLPP)

Stackelberg game is a leader–follower strategy and an N-people nonzero-sum game. In two-person nonzero-sum games [3], the objectives of the players are neither exactly opposite nor do they coincide with each other, and the loss of one of them is not equal to the other. According to the hierarchy of Stackelberg game with leader and follower, it can develop to become a multi-level programming problem. Based on the decomposition principle for linear programs [4], many organizations have rested heavily on the principle to optimize hierarchical systems. The assumption is that all variables have been controlled by centers, and the objectives of centers can decompose into the objectives of all subunits. However, some academics like Bialas and Karwan [5,6] argued that these problems are characterized by a hierarchy of planners, each independently controlling a set of decision variables, disjoint from the others.

BLPP is a special case of multi-level linear programming problems. Assume that the higher-level decision maker has control over X and lower-level decision maker has control over Y. Then, we have $x \in X \subset R^n$, $y \in Y \subset R^m$ and $F : X \times Y \to R^1$. The BLPP can be stated as follows:

$$\begin{aligned} \text{P1}: \quad & \min_{x \in X} \quad F(x,y) = c_1 x + d_1 y, \\ \text{P2}: \quad & \min_{y \in Y} \quad f(x,y) = c_2 x + d_2 y, \\ & \text{subject to } A_2 x + B_2 y \le b, \end{aligned} \tag{1}$$

where $c_1, c_2 \in R^n$, $d_1, d_2 \in R^m$, $b \in R^p$, $A \in R^{p\times n}$, $B \in R^{p\times m}$. $P1$ is the higher-level decision maker and $P2$ is the lower-level decision maker. According to the differences of the requirements in these models, there may be some extra limitations of x and y, like the limitation of integers or the limitation of upper and lower bound. In the process of solving, once the leader has chosen an x, the x of the follower's objective function has become a constant. Thus, the objective function of follower is simplified to $\min_{y\in Y} f(y) = d_2 y$.

Based on these we have the following definitions [7]:

Definition 1. 1. Constraint set of the problem:

$$S = \{(x,y) : x \in X, y \in Y, Ax + By \le b\}. \tag{2}$$

2. Feasible set for the follower for each x:

$$S(x) = \{y \in Y : By \le b - A\}. \tag{3}$$

3. Projection of M onto the leader's decision space:

$$S(x) = \{x \in X : \exists y \in Y, Ax + By \le b\}. \tag{4}$$

4. Follower rational reaction set for $x \in$ S (X):

$$P(x) = \{y \in Y : y \in \arg\min[f(\hat{x},\hat{y}) : \hat{y} \in S(x)]\}, \tag{5}$$

$$\text{and } \arg\min\{[f(\hat{x},\hat{y}) : \hat{y} \in S(x)]\} = \{f(x,y) \le f(x,\hat{y}), \hat{y} \in S(x)\}. \tag{6}$$

5. Inducible region:

$$IR = \{(x,y) : (x,y) \in S, y \in P(x)\}. \tag{7}$$

Definition 2. If $\{(x,y) \in P\ (\mathrm{x})\ | x \in S(X)\}$, (x,y) is the feasible solution of the BLPP.

BLPP is equivalent to a feasible region consisting of piecewise-linear constraints to minimize the objective function F of the leader.

Definition 3. For $\forall (x,y) \in IR$, if $\exists_{(x^*,y^*)} \in IR$, $F(x^*,y^*) \le F(x,y)$, then (x^*,y^*) is an optimal solution of problem.

To solve the BLPPs, it must be careful that when x on the upper level has been fixed, the corresponding solution on the lower level has not been the only one. There have been some effects on the objective function of the lower level, but it has had a large differentiation on the upper level. To overcome the problem, Bialas and Karwan [8] suggested replacing the f by $f + \varepsilon F$, where $\varepsilon > 0$ is a small factor. The idea is to have the upper decision maker share a small part of his earning with the decision maker, to make this latter choose the appropriate solution to the upper objective. In the organization, this can be regarded as sharing some profits of the upper manager to all departments to assure that all departments can work hard for the whole objectives in the company. In general, to check the effect of multiple optima, we can solve the BLPPs: once with the lower objective function equal to $f + \varepsilon F$, and a second time with that function equal to $f - \varepsilon F$. And optimal solution is obtained if it is optimal in both cases. Thus, it is the optimal solution.

The solution of BLPP may not be Pareto-optimal. In other words, it may be a feasible solution to get a better objective function of a lower or upper level but does not influence the objective function value of any other levels. Wen and Hsu [9]