

Pareto-optimal solutions in fuzzy multi-objective linear programming

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Abstract

The problem of solving multi-objective linear-programming problems, by assuming that the decision maker has fuzzy goals for each of the objective functions, is addressed. Several methods have been proposed in the literature in order to obtain fuzzy-efficient solutions to fuzzy multi-objective programming problems. In this paper we show that, in the case that one of our goals is fully achieved, a fuzzy-efficient solution may not be Pareto-optimal and therefore we propose a general procedure to obtain a non-dominated solution, which is also fuzzy-efficient. Two numerical examples illustrate our procedure.

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1. Introduction

Let a multi-objective linear programming (MOLP) problem with k objective functions $z_i(x) = c_i x$, $i = 1, \dots, k$ be

$$\text{Min } z(x) = (z_1(x), z_2(x), \dots, z_k(x))$$

$$\text{S.t. } x \in X, \tag{1}$$

where $X = \{x \in \mathbb{R}^n | Ax \geq b, x \geq 0\}$, $c_i = (c_{i1}, \dots, c_{in}) \in \mathbb{R}^n$, $i = 1, \dots, k$; $b = (b_1, \dots, b_m) \in \mathbb{R}^m$ and A is a $\mathbb{R}^{m \times n}$ matrix.

For the sake of simplicity we suppose that we want to minimize all the objective functions. However, the procedure demonstrated is easily extendable were the case to include some maximizing objectives.

In problem (1) it is unlikely that all objective functions will simultaneously achieve their optimal value. Therefore in practice the Decision maker (DM) chooses a satisficing solution, according to the aspiration level fixed for each objective.

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Assuming that the DM proposes imprecise aspiration levels such as, “the objective function $z_i(x)$ should be essentially less than or equal to some value g_i ”, model (1) can be written as

$$\begin{aligned} &\text{Find } x \\ &\text{Such that } z_i(x) \lesssim g_i \quad i = 1, 2, \dots, k \\ &x \in X. \end{aligned} \quad (2)$$

Each expression $c_i x \lesssim g_i$ is represented by a fuzzy set called fuzzy goal, whose membership function, $\mu_i(z_i)$, $\mu_i : \mathbb{R} \rightarrow [0, 1]$, provides the satisfaction degree λ_i to which the i th fuzzy inequality is satisfied. In order to define the membership function $\mu_i(z_i)$ the DM has to provide the tolerance margins $g_i + t_i$ that he is willing to accept. So $\mu_i(z_i)$ should be equal to 1 if $z_i \leq g_i$, strictly monotone decreasing from 1 to 0 over the interval $(g_i, g_i + t_i)$ and equal to 0 if $z_i \geq g_i + t_i$.

Models like (2) are named fuzzy multi-objective linear programming (FMOLP) problems. As it has been widely seen, a FMOLP problem, using the fuzzy decision (max–min) of Bellman and Zadeh [1] and introducing the auxiliary variable λ adopts the following formulation [13]:

$$\begin{aligned} &\text{Max } \lambda \\ &\text{S.t. } 1 \geq \mu_i(z_i(x)) \geq \lambda \geq 0, \quad i = 1, \dots, k, \\ &x \in X. \end{aligned} \quad (3)$$

If there exists a unique optimal solution of (3), then it is a fuzzy-efficient solution to the FMOLP problem (2) (see Definition 1). However if the uniqueness of a solution is not satisfied, the fuzzy-efficiency is not guaranteed for all solutions of model (3), but at least one of the multiple optimal solutions is fuzzy-efficient [11]. In order to produce a fuzzy-efficient solution several approaches have been proposed in the literature. Guu and Wu [3,4], proposed a second step in which they use the additive criterion to aggregate the fuzzy goals. Sakawa et al. [9,10] proposed an interactive procedure. However, during some stages of this process they use the max–min approach whereas during other stages they use the additive criterion. As Dubois and Fortemps [2] say these kinds of approaches are partially inconsistent because they change from a non compensatory criterion (min) to a compensatory criterion (sum) during different stages. In order to overcome this drawback Dubois and Fortemps [2] propose a multi-step procedure which involves solving sequentially, several max–min optimization problems.

In order to illustrate our procedure, for the sake of simplicity, in this paper we only put emphasis on the procedures suggested by Guu and Wu and Dubois and Fortemps, but any other procedure that supplies a fuzzy-efficient solution could be used.

Indeed, in this paper we propose an extension to the Guu and Wu and Dubois and Fortemps approaches. Our method finds an efficient solution to a more general case than that of the aforementioned authors. In order to achieve this, we use a modified technique for generating a Pareto-optimal solution to the MOLP problem (1) which preserves the property of being a fuzzy-efficient solution to the FMOLP problem (2).

The paper is organized as follows. In Section 2, we show that a fuzzy-efficient solution for the FMOLP model (2), in which one of the goals is fully achieved, may or may not be a Pareto-optimal solution for the MOLP model (1). We then propose a method that improves the aforementioned fuzzy-efficient solution generating a Pareto-optimal solution that preserves the property of being fuzzy efficient. Finally, in Section 3, we show how our method can be used as an extension to several published procedures and, to further explain our approach, we solve two numerical examples, the second example being the same as that suggested by Guu and Wu [4] in illustrating their method.

2. A model to obtain Pareto-optimal solutions in case some goals are fully achieved

Within the scope of the multi-criteria decision making (MCDM) theory, the Pareto-optimality is a necessary condition in order to guarantee the rationality of a decision. Therefore a “reasonable” solution to a MOLP problem should be Pareto-optimal and, as FMOLP is an approach which is used in order to be able to solve MOLP problems, any optimal solution of a FMOLP problem should be also Pareto-optimal.

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