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Bector–Chandra type duality in fuzzy linear programming with exponential membership functions

Pankaj Gupta*, Mukesh Kumar Mehlawat

Department of Operational Research, University of Delhi, Delhi 110007, India

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Abstract

In this paper, we study a pair of fuzzy primal–dual linear programming problems and calculate duality results using an aspiration level approach. We use an exponential membership function, which is in contrast to the earlier works that relied on a linear membership function. As the fuzzy environment causes a duality gap, we investigate how choosing the exponential membership function impacts this gap. This issue is particularly important for fuzzy linear programming where, in general, the primal and dual objective values may not be bounded.

Keywords: Fuzzy mathematical programming; Fuzzy constraint satisfaction; Primal–dual problems; Duality theory

1. Introduction

Linear programming is one of the most widely used decision making tools for solving real world problems. One of the main assumptions used in this technique is that the input data have complete accuracy. However, more often than not, real world situations are characterized by imprecision (fuzziness) rather than exactness. Therefore, a number of researchers have shown interest in the area of fuzzy linear programming. Fuzzy linear programming can be classified on the basis of two concepts: (i) the limits of fuzziness that the decision maker sets—the decision maker's aspirations—with respect to the objective and/or constraints and (ii) the ambiguity in the coefficients of the objective function and/or constraints. The combination of these two concepts gives us different types of fuzzy linear programming problems. Since a given linear programming method does not admit a unique type of fuzzy linear programming, it can hardly be expected that there is a unique duality theory for fuzzy linear programming. Most of the duality results in fuzzy linear programming have been obtained by using either an aspiration level approach or a method that allows general fuzzy (valued) relations to be specified.

In this paper, we attempt to generate duality results for a pair of fuzzy primal–dual linear programming problems using the aspiration level approach described by Zimmermann [36]. The aspiration level approach used in the present study is based on the fact that in practice a decision maker is more comfortable describing fuzzy constraints or establishing aspiration levels for the objective and/or constraints than specifying a large number of fuzzy numbers for the various

* Corresponding author. Flat No.-01, Kamayani Kunj, Plot No. 69, I.P. Extension, Delhi 110092, India.
E-mail address: pankaj_gupta15@yahoo.com (P. Gupta).

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parameters of the problem. Moreover, the aspiration level approach is more effective as it does not require consistency in the decision maker's judgment; the aspiration levels are more like a probe than weighting parameters.

To understand duality in fuzzy linear programming using an aspiration level approach, we may refer to the work of Rödder and Zimmermann [23]. They started from the saddle point property of the Lagrangian function

$$c^t x + w_0^t(b - Ax) \leq c^t x_0 + w_0^t(b - Ax_0) \leq c^t x_0 + w^t(b - Ax_0),$$

where x and w are the primal and dual variables, respectively, that soften the strict "min–max" imperative. If x and w are considered to be the decision variables of the two competitors in an oligopolistic market, duality theory for linear programming only considers the optimal behavior of each competitor. When fuzzyfying the structure analyzed with linear programming, suboptimal (almost optimal) behavior can be considered using special types of membership functions.

The present research relies on the fact that the duality between fuzzy primal–dual linear programming problems can be relaxed (with additional terms) from the conventional duality (without additional terms) owing to the presence of fuzzy constraints (soft constraints) as in the work of Bector and Chandra [1]. However, the duality results generated with a fuzzy environment must conform to the corresponding results for the crisp situations; for reference, see [1]. This implies that *the duality results of linear programming problem for a fuzzy environment are a special case and lead to the corresponding results from crisp situations*. The concept of duality for a fuzzy environment used in the present study is well supported by a significant amount of prior research, e.g. Hamacher et al. [9], Rödder and Zimmermann [23], Liu et al. [16], Bector and Chandra [1,4], Bector et al. [2,3], Vijay et al. [29].

It is important to note that when implementing fuzzy linear programming formulations based on the vague aspiration levels of the decision maker, the objective function and the system constraints are defined by a unique membership function. This membership function acts as a surrogate characterization of the operator's preferences for determining the desired outcome of the objective. Several membership functions have been employed in fuzzy linear programming: (i) linear [36], (ii) piecewise linear [11], (iii) exponential [7,8,15,28], (iv) hyperbolic [14], (v) logistic [24], and (vi) S-shaped [26]. This list is by no means intended to represent the entirety of membership functions in existence. A linear membership function is most commonly used because it is simple and it is defined by fixing two points: the upper and lower levels of acceptability. However, a linear membership function is not a suitable representation in many practical situations [10]. Furthermore, if the membership function is interpreted as the fuzzy utility of the decision maker, used for describing levels of indifference, preference or aversion towards uncertainty, then a nonlinear membership function provides a better representation than a linear membership function. Moreover, it should be emphasized that unlike linear membership functions, for nonlinear membership functions the marginal rate of increase (or decrease) of membership values as a function of model parameters is not constant—a technique that reflects reality better than the linear case.

In view of the discussion above, it is desirable that nonlinear membership functions are used more frequently in fuzzy linear programming. It is notable that one can obtain duality results even by using piecewise linear/hyperbolic/logistic/S-shaped membership functions. However, the results thus obtained would not conform to the corresponding duality results from crisp situations. Hence, in this paper, we choose exponential membership functions to establish duality results for a fuzzy environment. Our results not only conform better to the corresponding duality results for the crisp situations, but they also fully incorporate the aspiration levels/tolerances for the objective function and the system constraints indicated by the decision maker. An important issue that has yet to be resolved is the duality gap that arises from the fuzzy environment. In the present work, this issue has been addressed by examining whether the duality gap can be altered by varying the shape parameters of the exponential membership functions.

The purpose of this paper is thus twofold. First, we show that the duality results (similar to Bector and Chandra [1]) can be produced by using exponential membership functions in place of linear membership functions. Second, we illustrate—with the help of a numerical example—that for a given set of aspiration levels and allowable tolerances, the duality gap from the fuzzy environment can be altered by selecting various shape parameters.

This paper is organized as follows. Section 2 is a literature review of duality in fuzzy linear programming. In Section 3, we consider a pair of fuzzy primal–dual linear programming problems in which vague aspiration levels are represented by exponential membership functions. We establish appropriate duality results for a fuzzy environment. In Section 4, we present numerical illustrations of the duality results and related situations. In this section, we also compare the duality gap in the present study with the gap reported in [1] and discuss its application. In Section 5, we extend our approach to fuzzy linear programming problems with fuzzy parameters and fuzzy constraints. We also consider some