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A note on "Solving linear programming problems under fuzziness and randomness environment using attainment values"

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ABSTRACT

This paper is an amendment to Hop's paper [N.V. Hop, Solving linear programming problems under fuzziness and randomness environment using attainment values, Information Sciences 177 (2007) 2971–2984], in solving linear programming problems under fuzziness and randomness environments. Hop introduced a new characterization of relationship, attainment values, to enable the conversion of fuzzy (stochastic) linear programming models into corresponding deterministic linear programming models. The purpose of this paper is to provide a correction and an improvement of Hop's analytical work through rationalization and simplification. More importantly, it is shown that Hop's analysis does not support his demonstration or the solution-finding mechanism; the attainment values approach as he had proposed does not result in superior performance as compared to other existing approaches because it neglects some relevant and inevitable theoretical essentials. Two numerical examples from Hop's paper are also employed to show that his approach, in the conversion of fuzzy (stochastic) linear programming problems to corresponding problems, is questionable and can neither find the maximum nor the minimum in the examples. The models of the examples are subsequently amended in order to derive the correct optimal solutions.

1. Introduction

In view of its properties that is applicable to real-life business applications, fuzzy linear programming has attracted a great deal of attention from researchers. Among these investigations [2,21,3,10,17] the most widely-adopted procedure to find the solution was through the conversion of fuzzy linear programming models into corresponding deterministic linear programming models [8]. For instance, Leung [11] tried to classify fuzzy mathematical programming models into four categories: a precise objective with fuzzy constraints, a fuzzy objective with precise constraints, a fuzzy objective with fuzzy constraints, and robust programming. In trying to find a solution to the problem, the procedure in each of the categories were also developed which includes the signed distance method [3], the area compensation method [4], the expected mid-point approach [10] and the grade of possibility and necessity measures [1]. Zimmermann [22] is viewed as the pioneer in solving linear programming problems with fuzzy resources and fuzzy objectives. He developed a max–min tolerance method with the criteria of the highest membership degree to convert the initial fuzzy linear programming models into corresponding crisp ones. A unique solution could then be found by using the Simplex method. A number of researchers have developed

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various approaches with different degrees of efficiency and effectiveness, but most were built upon Zimmermann's max–min approach.

As for extending the research of fuzzy optimization, a new direction in the investigation focuses on a fuzzy and stochastic environment. Solving fuzzy stochastic linear programming problems, therefore, becomes important. In essence, fuzzy numbers/variables and stochastic variables are considered to be a more suitable characterization for real-world problems where uncertain and imprecise information is inherent. However, the inclusion of those components creates a challenge for finding efficient methods to deal with such conditions. An effective way to handle the fuzzy stochastic optimization problems is to convert the problems by de-fuzzifying the fuzzy numbers/variables, de-randomizing the stochastic variables and to solve the resulting deterministic problems instead.

Two main approaches were established to cope with the fuzzy stochastic problems. One is to perform the conversion of the de-fuzzification and de-randomization in a sequential manner [16,15,9]. The other is to perform both processes in a simultaneous manner [6,12,14]. As for the sequential approach, a major disadvantage is that it generates an excessive amount of new variables and constraints to the model. On the other hand, a major disadvantage of the simultaneous approach is having a cumbersome workload for calculating the expected value for removing fuzziness and randomness at the same time.

In the pursuit of improving the performance of existing approaches, Hop has established a new approach that enables a reduction in the number of additional constraints and an achievement to a certain degree of computational efficiency. The primary feature of Hop's proposed approach is that instead of using the absolute relationship, as applied in the signed distance method, he employed the relative relationship between fuzzy numbers and fuzzy stochastic variables. The relative relationship is obtained through the calculation of so-called "attainment values" or degrees such as lower-side attainment index, both-side attainment index and average index as mentioned in his study. The relative relationship and thus the attainment values, play the key role in the conversion of fuzzy and fuzzy stochastic linear programming problems into more simple, conventional crisp problems, while reducing the number of constraints and computational complexity. Hop emphasized that his converted deterministic LP with few additional constraints can be easily solved by using standard optimization packages such as LINGO. Two examples were provided to illustrate the procedure of his proposed approach.

However, we can verify that the method of attainment values that Hop [7] proposed is flawed since it can neither find the maximum nor the minimum values of the desired objectives owing to negligence of some relevant and unavoidable theoretical essentials. To the best of our knowledge, at least four papers [5,18–20], thus far have referred to Hop [7] in their references. Of them, none have noticed the flawed results established and claimed in Hop's work. Here we have proceeded with a thorough investigation of Hop's paper and highlighted mistakes in his conversion and his solution-finding procedures in an effort to alert future adoption of this particular solution approach in fuzzy/fuzzy stochastic programming and related areas.

The remainder of this paper is organized as follows. Section 2 reviews the mathematical formulation of Hop's research. Section 3 provides our revision necessary in order to resolve the problems in Hop's approach. Section 4 reconsiders the two numerical examples of Hop's investigation and compares these results with those from our approach. Finally, conclusions are drawn and future work outlined.

2. Review of Hop's results in mathematical formulation

Hop [7] tried to provide a new approach to solve linear programming problems under fuzzy and random conditions, and that is without the complicated computation of expected values as appeared in Liu and Liu [13]. To achieve this objective, the attainment indices are defined and utilized. For two fuzzy numbers $\widetilde{P}$ and $\widetilde{Q}$, with $\widetilde{P} \leqslant \widetilde{Q}$, the lower-side attainment index of $\widetilde{P}$ to $\widetilde{Q}$ is defined by Hop [7] as

$$D(\widetilde{P},\widetilde{Q}) = \int_0^1 \max\left\{0, \sup\{s : \widetilde{P}(s) \geqslant \alpha\} - \inf\{r : \widetilde{Q}(r) \geqslant \alpha\}\right\} d\alpha. \tag{1}$$

For triangular fuzzy number $\widetilde{P} = (u, a, b)$ with the membership function $\mu_{\widetilde{P}}(x)$, we rewrite the membership function in a more compact expression.

$$\mu_{\widetilde{P}}(x) = \begin{cases} \max\left\{0, 1 - \frac{u-x}{a}\right\}, & \text{if } x \leqslant u, \\ \max\left\{0, 1 - \frac{x-u}{b}\right\}, & \text{if } x > u. \end{cases} \tag{2}$$

Proposition 1 of Hop [7]. *For two triangular fuzzy numbers, $\widetilde{P} = (u, a, b)$ and $\widetilde{Q} = (v, c, d)$ with $u \leqslant v$, the average lower-side attainment index of $\widetilde{P}$ to $\widetilde{Q}$ is defined as $\overline{D}(\widetilde{P},\widetilde{Q}) = \frac{u-v+b+c}{2}$.*

Proposition 1 as proved by Hop is as follows.

The α-cut of $\widetilde{P}$ is defined as $[P^l(\alpha), P^u(\alpha)] = [u - a(1-\alpha), u + b(1-\alpha)]$, and of $\widetilde{Q}$ is $[Q^l(\alpha), Q^u(\alpha)] = [v - c(1-\alpha), v + d(1-\alpha)]$. Hop [7] had the lower-side attainment index of fuzzy number $\widetilde{P}$ to $\widetilde{Q}$ at α-level:

$$D(\widetilde{P},\widetilde{Q})_\alpha = \max\{0, P^u(\alpha) - Q^l(\alpha)\} = \max\{0, u - v + (b+c)(1-\alpha)\}. \tag{3}$$