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Linear programming method for multiattribute group decision making using IF sets

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ABSTRACT

The purpose of this paper is to develop a linear programming methodology for solving multiattribute group decision making problems using intuitionistic fuzzy (IF) sets. In this methodology, IF sets are constructed to capture fuzziness in decision information and decision making process. The group consistency and inconsistency indices are defined on the basis of pairwise comparison preference relations on alternatives given by the decision makers. An IF positive ideal solution (IFPIS) and weights which are unknown *a priori* are estimated using a new auxiliary linear programming model, which minimizes the group inconsistency index under some constraints. The distances of alternatives from the IFPIS are calculated to determine their ranking order. Moreover, some properties of the auxiliary linear programming model and other generalizations or specializations are discussed in detail. Validity and applicability of the proposed methodology are illustrated with the extended air-fighter selection problem and the doctoral student selection problem.

1. Introduction

Multiattribute decision making (MADM) problems can be solved using several existing methods such as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [22] and the Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP) [33]. The TOPSIS and the LINMAP are two well-known MADM methods, though they require different types of information and decision conditions [11–13,21,30,32].

In the LINMAP, all the decision data are known precisely or given as numeric values. However, under many conditions, numeric values are inadequate or insufficient to model real-life decision problems [1,2,9,11–20,22–26,28,31,34,36,37]. Indeed, human judgments including preference information are vague or fuzzy in nature and as such it may not be appropriate to represent them by accurate numeric values. Since the concept of the fuzzy set was introduced by Zadeh [40] in 1965, the fuzzy set theory has been used to handle MADM problems and has achieved a great success [1,2,8,10–12,35–38,41,42]. In the fuzzy set theory, the degree of membership for an element x is $\mu(x)$ and the degree of non-membership is $1-\mu(x)$ automatically, i.e., this membership degree combines the evidence for x and the evidence against x. The single number tells us nothing about the lack of knowledge. In real applications, however, information of an element belonging to a fuzzy concept may be incomplete, i.e., the sum of the membership degree and the non-membership degree may be less than one. There is no means to incorporate the lack of knowledge of the membership degree in the fuzzy set. A possible solution is to use the intuitionistic fuzzy (IF) set introduced by Atanassov [3–6], which is a generalization of the fuzzy set. The reason is that the IF set

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seems to be well suited for expressing hesitation of the decision makers (or experts) [7,27,29,39]. Therefore, the purpose of this paper is to further extend the LINMAP to develop a new methodology for solving multiattribute group decision making (MAGDM) problems using IF sets. In this methodology, IF sets are used to describe fuzziness in decision information and decision making process by means of IF decision matrices. An IF positive ideal solution (IFPIS) [43–45] and weights of attributes are estimated using a new auxiliary linear programming model based upon the group consistency and inconsistency indices, which are defined on the basis of pairwise comparison preference relations on alternatives given by the decision makers. The distances of alternatives to the IFPIS are calculated to determine their ranking orders for the decision makers. The ranking order of alternatives for the group can be generated using the Borda's function [22,24].

The rest of this paper is organized as follows. In Section 2, the concept of the IF set and the Euclidean distance between IF sets are introduced and the MAGDM problems using IF sets are formulated. In Section 3, the group consistency and inconsistency indices are defined. A new auxiliary linear programming model is constructed to estimate the IFPIS and attribute weights which are unknown *a priori*. Some properties of the auxiliary linear programming model and other generalizations or specializations are discussed in detail. The proposed methodology is illustrated with a numerical example of the extended air-fighter selection problem and compared with other similar methods in Section 4. A practical application of the proposed methodology is shown with the doctoral student selection problem in Section 5. Further discussions on the proposed methodology and conclusions are given in Sections 6 and 7, respectively.

2. Basic concepts and notations

2.1. The concept of the IF set

The concept of the IF set was firstly introduced by Atanassov [3,4] in 1983. Let $U = \{z_l | l = 1, 2, \ldots, L\}$ denote a finite universal of discourse. An IF set V in U is an object having the following form:

$$V = \{\langle z_l, \mu_V(z_l), \upsilon_V(z_l)\rangle | z_l \in U\},$$

where the functions

$$\mu_V : U \mapsto [0, 1]$$
$$z_l \in U \to \mu_V(z_l) \in [0, 1]$$

and

$$\upsilon_V : U \mapsto [0, 1]$$
$$z_l \in U \to \upsilon_V(z_l) \in [0, 1]$$

define the degree of membership and the degree of non-membership of an element $z_l \in U$ to the set $V \subseteq U$, respectively, and such that they satisfy the following condition: $0 \leqslant \mu_V(z_l) + \upsilon_V(z_l) \leqslant 1$ for every $z_l \in U$. From the practical viewpoint, the membership degree and the non-membership degree in IF sets are exact without any assumption on indeterminacy, except for $\mu_V(z_l) + \upsilon_V(z_l) \leqslant 1$, and more or less independent.

Let

$$\pi_V(z_l) = 1 - \mu_V(z_l) - \upsilon_V(z_l),$$

which is called the IF index of an element z_l in the set V. It is the degree of indeterminacy membership of the element z_l to the set V. Obviously, $0 \leqslant \pi_V(z_l) \leqslant 1$.

If $\mu_V(z_l) + \upsilon_V(z_l) = 1$, i.e., $\pi_V(z_l) = 0$, then V degenerates to a fuzzy set, which may be written as $V = \{\langle z_l, \mu_V(z_l), 1 - \mu_V(z_l)\rangle | z_l \in U\}$. Conversely, a fuzzy set $V = \{\langle z_l, \mu_V(z_l)\rangle | z_l \in U\}$ can be written as an IF set, $V = \{\langle z_l, \mu_V(z_l), 1 - \mu_V(z_l)\rangle | z_l \in U\}$.

If $\mu_V(z_l)$ is equal to 1 or 0, then V degenerates to a crisp set, i.e., $V = \{\langle z_l, 1, 0\rangle | z_l \in U\}$ or $V = \{\langle z_l, 0, 1\rangle | z_l \in U\}$. Conversely, a crisp set V can be written as an IF set, $V = \{\langle z_l, \chi_V(z_l), 1 - \chi_V(z_l)\rangle | z_l \in U\}$, where

$$\chi_V(z_l) = \begin{cases} 1 & \text{if } z_l \in V, \\ 0 & \text{if } z_l \notin V. \end{cases}$$

2.2. The Euclidean distance between IF sets

The concept of the distance between IF sets was firstly introduced by Atanassov [3,4]. Let V_1 and V_2 be two IF sets in the set U. Namely, $V_1 = \{\langle z_l, \mu_{V_1}(z_l), \upsilon_{V_1}(z_l)\rangle | z_l \in U\}$ and $V_2 = \{\langle z_l, \mu_{V_2}(z_l), \upsilon_{V_2}(z_l)\rangle | z_l \in U\}$, respectively, where $\pi_{V_1}(z_l) = 1 - \mu_{V_1}(z_l) - \upsilon_{V_1}(z_l)$ and $\pi_{V_2}(z_l) = 1 - \mu_{V_2}(z_l) - \upsilon_{V_2}(z_l)$. The Euclidean distance between V_1 and V_2 is defined as follows:

$$d(V_1, V_2) = \sqrt{(1/2)\sum_{l=1}^{L}[(\mu_{V_1}(z_l) - \mu_{V_2}(z_l))^2 + (\upsilon_{V_1}(z_l) - \upsilon_{V_2}(z_l))^2 + (\pi_{V_1}(z_l) - \pi_{V_2}(z_l))^2]}. \tag{1}$$