Expert Systems with Applications 37 (2010) 5939–5945

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Linear programming method for MADM with interval-valued intuitionistic fuzzy sets

Deng-Feng Li *

School of Management, Fuzhou University, Fuzhou 350108, Fujian, China
Department Five, Dalian Naval Academy, Dalian 116018, Liaoning, China

ARTICLE INFO

Keywords:
Multiattribute decision making
Intuitionistic fuzzy set
Interval-valued intuitionistic fuzzy set
Mathematical programming
Uncertainty

ABSTRACT

Fuzziness is inherent in decision data and decision making process. In this paper, interval-valued intuitionistic fuzzy (IVIF) sets are used to capture fuzziness in multiattribute decision making (MADM) problems. The purpose of this paper is to develop a methodology for solving MADM problems with both ratings of alternatives on attributes and weights being expressed with IVIF sets. In this methodology, a weighted absolute distance between IF sets is defined using weights of IF sets. Based on the concept of the relative closeness coefficients, we construct a pair of nonlinear fractional programming models which can be transformed into two simpler auxiliary linear programming models being used to calculate the relative closeness coefficient intervals of alternatives to the IVIF positive ideal solution, which can be employed to generate ranking order of alternatives based on the concept of likelihood of interval numbers. The proposed method is illustrated with a real example.

1. Introduction

The theory of the fuzzy set introduced by Zadeh (1965) has achieved a great success in various fields. Atanassov (1986, 1999) introduced the intuitionistic fuzzy (IF) set, which is a generalization of the fuzzy set. Gau and Buehrer (1993) introduced the concept of the vague set, which is another generalization of the fuzzy set. But, it was proven that the vague set is the same as the IF set (Burillo & Bustince, 1996). The IF set has received more and more attention and has been applied to many fields since its appearance. The theory of the IF set has been found to be more useful to deal with vagueness and uncertainty in decision situations than that of the fuzzy set (Atanassov, Pasi, & Yager, 2005; Deschrijver & Kerre, 2007; Szmidt & Kacprzyk, 1996a, 1996b, 1996c, 1997, 2002). Over the last decades, the IF set theory has been successfully applied to solve decision making problems (Chen & Tan, 1994; Hong & Choi, 2000; Li, 2005, 2008; Liu & Wang, 2007, 2008; Li, Wang, Liu, & Shan, 2009; Pankowska & Wygralak, 2006; Szmidt & Kacprzyk, 1996a, 1996b, 1996c, 1997, 2002; Xu, 2007a, 2007b, 2007c, 2007d; Xu & Yager, 2006). Atanassov and Gargov (1989) further generalized the IF set in the spirit of ordinary interval-valued fuzzy (IVF) sets and defined the notion of an interval-valued intuitionistic fuzzy (IVIF) set. The relations, operations and operators related to IF sets and IVIF sets have been systematically studied in (Deschrijver & Kerre, 2003).

Recently, some researchers proposed several aggregation operators such as the IF weighted averaging operator, the IVIF weighted averaging operator, the IF ordered weighted averaging operator, the IVIF ordered weighted averaging operator and the IF ordered weighted geometric operator as well as the IVIF ordered weighted geometric operator, and employed them to deal with multiattribute decision making (MADM) with IF and IVIF information (Xu, 2007d, 2007e; Xu & Chen, 2007; Xu & Yager, 2006). Ye (2009) introduced the IVIF weighted arithmetic average operator, the IVIF weighted geometric average operator and a novel accuracy function of IVIF values. However, there exist little investigation on MADM problems with both ratings of alternatives on attributes and weights being expressed with IVIF sets. In this paper, a weighted absolute distance between IF sets is defined using weights of IF sets. Then, based on the concept of the relative closeness coefficients, a pair of nonlinear fractional programming models is constructed to calculate the relative closeness coefficient intervals of alternatives with respect to the IVIF positive ideal solution (IVIFPIS), which can be used to generate ranking order of the alternatives. The nonlinear fractional programming models can be transformed into two auxiliary linear programming models, respectively.

The paper is organized as follows. Section 2 briefly introduces the concept of the IF set and the IVIF set. A weighted absolute distance between IF sets and the definition of likelihood for

* Address: Department Five, Dalian Naval Academy, No. 1, Xiaolong Street, Dalian 116018, Liaoning, China. Tel.: +86 0411 85856357.
E-mail addresses: dengfengli@sina.com, lidengfeng65@hotmail.com

0957-4174/$ - see front matter
doi:10.1016/j.eswa.2010.02.011

comparison between two interval numbers are given. The MADM problem with IVIF sets is presented in Section 3. In Section 4, a pair of nonlinear fractional programming models is constructed based on the concept of the relative closeness coefficients and is transformed into two simpler auxiliary linear programming models to solve the MADM problems with IVIF sets. A real example and short remark are given in Sections 5 and 6, respectively.

2. Interval-valued intuitionistic fuzzy sets and ranking of interval numbers

Atanassov (1986, 1999) firstly introduced the concept of the IF set.

Definition 1 (Atanassov, 1986, 1999). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. An IF set A in X is an object having the following form:

$$A = \{\langle x_j, \mu_A(x_j), \upsilon_A(x_j)\rangle | x_j \in X\},$$

where the functions

$$\mu_A : X \mapsto [0,1],$$
$$x_j \in X \to \mu_A(x_j) \in [0,1]$$

and

$$\upsilon_A : X \mapsto [0,1],$$
$$x_j \in X \to \upsilon_A(x_j) \in [0,1]$$

define the degree of membership and degree of non-membership of the element $x_j \in X$ to the set $A \subseteq X$, respectively, and for every $x_j \in X, 0 \leqslant \mu_A(x_j) + \upsilon_A(x_j) \leqslant 1$.

Let

$$\pi_A(x_j) = 1 - \mu_A(x_j) - \upsilon_A(x_j),$$

which is called the IF index of the element x_j in the set A. It is the degree of indeterminacy membership of the element x_j to the set A. Obviously,

$$0 \leqslant \pi_A(x_j) \leqslant 1.$$

The concept of distances between IF sets was firstly proposed by Atanassov (1999). Other researchers also proposed some distance formulae (Grzegorzewski, 2004; Szmidt & Kacprzyk, 2001; Wang & Xin, 2005). In this paper, the following weighted absolute distance between IF sets is proposed. Assume that $C = \{\langle x_j, \mu_C(x_j), \upsilon_C(x_j)\rangle | x_j \in X\}$ be an IF set of importance of all elements $x_j \in X$. A weighted absolute distance between $A = \{\langle x_j, \mu_A(x_j), \upsilon_A(x_j)\rangle | x_j \in X\}$ and $B = \{\langle x_j, \mu_B(x_j), \upsilon_B(x_j)\rangle | x_j \in X\}$ is defined as follows

$$d(A,B) = \sum_{j=1}^{n} [\mu_C(x_j)|\mu_A(x_j) - \mu_B(x_j)| + \upsilon_C(x_j)|\upsilon_A(x_j) - \upsilon_B(x_j)|]. \quad (1)$$

Definition 2 (Atanassov and Gargov, 1989). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set and I be the set of all closed subintervals of the interval $[0,1]$. An IVIF set D in X is an object having the following form:

$$D = \{\langle x_j, \tilde{\mu}_D(x_j), \tilde{\upsilon}_D(x_j)\rangle | x_j \in X\},$$

where the functions

$$\tilde{\mu}_D : X \mapsto I,$$
$$x_j \in X \to \tilde{\mu}_D(x_j) \subseteq [0,1]$$

and

$$\tilde{\upsilon}_D : X \mapsto I,$$
$$x_j \in X \to \tilde{\upsilon}_D(x_j) \subseteq [0,1]$$

define the intervals of the degree of membership and degree of non-membership of the element $x_j \in X$ to the set $D \subseteq X$, respectively, and for every $x_j \in X, 0 \leqslant \sup\{\tilde{\mu}_D(x_j)\} + \sup\{\tilde{\upsilon}_D(x_j)\} \leqslant 1$.

Obviously, $\tilde{\mu}_D(x_j)$ and $\tilde{\upsilon}_D(x_j)$ are closed intervals. Their lower and upper bounds are denoted by $\mu_D^l(x_j), \mu_D^u(x_j), \upsilon_D^l(x_j)$ and $\upsilon_D^u(x_j)$, respectively. Thus, the IVIF set D may be expressed as follows

$$D = \{\langle x_j, [\mu_D^l(x_j), \mu_D^u(x_j)], [\upsilon_D^l(x_j), \upsilon_D^u(x_j)]\rangle | x_j \in X\}.$$

To rank order among interval numbers, the concept of likelihood is introduced in the following. Denote $a \geqslant b$, which means "a being not smaller b". Assume that $a = [a^-, a^+]$ and $b = [b^-, b^+]$ be any two interval numbers, denote their interval lengths by $L(a) = a^+ - a^-$ and $L(b) = b^+ - b^-$, respectively. Obviously, $a = [a^-, a^+]$ may degenerate to a real number $\bar{a}$ if $a^- = a^+$, where $\bar{a} = a^- = a^+$.

Definition 3. (Xu and Da, 2003). For any two real numbers a and b, the likelihood of $a > b$ is defined as follows

$$p(a > b) = \begin{cases} 1, & a < b, \\ 0, & a \geqslant b. \end{cases}$$

Definition 4. (Xu and Da, 2003). For any two interval numbers $a = [a^-, a^+]$ and $b = [b^-, b^+]$, the likelihood of $a \geqslant b$ is defined as follows

$$p(a \geqslant b) = \max\left\{1 - \max\left\{\frac{b^+ - a^-}{L(a) + L(b)}, 0\right\}, 0\right\}, \quad (2)$$

where $L(a) = a^+ - a^-$ and $L(b) = b^+ - b^-$.

The likelihood of $a \geqslant b$ for any two interval numbers a and b has some useful properties which are summarized in the following.

(a) $0 \leqslant p(a \geqslant b) \leqslant 1$;
(b) $p(a \geqslant b) + p(b \geqslant a) = 1$;
(c) $p(a \geqslant b) = p(b \geqslant a) = 1/2$ if $p(a \geqslant b) = p(b \geqslant a)$;
(d) $p(a \geqslant b) = 0$ if $a^+ \leqslant b^-$;
(e) For any interval numbers a, b and $c, p(a \geqslant c) \geqslant p(b \geqslant c)$ if $a \geqslant b$.

The above properties (a)–(e) are easily proved (omitted).

3. MADM problems with IVIF sets

Assume that $[\mu_{ij}^l, \mu_{ij}^u]$ and $[\upsilon_{ij}^l, \upsilon_{ij}^u]$ be intervals of the degrees of membership and the degrees of non-membership of alternatives $x_i \in X$ on attributes $a_j \in A$ with respect to the concept "excellence", respectively, where $0 \leqslant \mu_{ij}^l \leqslant \mu_{ij}^u \leqslant 1, 0 \leqslant \upsilon_{ij}^l \leqslant \upsilon_{ij}^u \leqslant 1$ and $\mu_{ij}^u + \upsilon_{ij}^u \leqslant 1$. In other words, ratings of the alternatives $x_i \in X$ on attributes $a_j \in A$ are IVIF sets, denoted by $X_{ij} = \{\langle x_i, [\mu_{ij}^l, \mu_{ij}^u], [\upsilon_{ij}^l, \upsilon_{ij}^u]\rangle\}$. For short, denote $X_{ij} = \langle [\mu_{ij}^l, \mu_{ij}^u], [\upsilon_{ij}^l, \upsilon_{ij}^u]\rangle$. Thus, a MADM problem with IVIF sets can be expressed concisely in the interval-valued matrix format as follows