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Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach

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ABSTRACT

In this paper, two new algorithms are presented to solve *multi-level multi-objective linear programming (ML-MOLP) problems* through the fuzzy goal programming (FGP) approach. The membership functions for the defined fuzzy goals of all objective functions at all levels are developed in the model formulation of the problem; so also are the membership functions for vectors of fuzzy goals of the decision variables, controlled by decision makers at the top levels. Then the fuzzy goal programming approach is used to achieve the highest degree of each of the membership goals by minimizing their deviational variables and thereby obtain the most satisfactory solution for all decision makers.

The first suggested algorithm groups the membership functions for the defined fuzzy goals of the objective functions at all levels and the decision variables for each level except the lower level of the multi-level problem. The second proposed algorithm lexicographically solves MOLP problems of the ML-MOLP problem by taking into consideration the decisions of the MOLP problems for the upper levels. An illustrative numerical example is given to demonstrate the algorithms.

1. Introduction

The standard mathematical programming problem involves finding an optimal solution for just one decision maker. Nevertheless, many planning problems contain an hierarchical decision structure, each with independent, and often conflicting objectives. These types of problems can be modeled using a multi-level mathematical programming (MLMP) approach. The basic concept of the MLMP technique is that the first-level decision maker (FLDM) sets his/her goal and/or decision, and then asks each subordinate level of the organization for their optima, that calculated in isolation. The lower level decision makers' decisions are then submitted and modified by the FLDM in consideration of the overall benefit for the organization. The process continues until a satisfactory solution is reached.

Most of the developments in MLP problems focus on bi-level linear programming as a class of MLP [1–4]. Bi-level nonlinear programming was studied in [5,6]. In [7], an interactive algorithm for bi-level multi-objective programming is presented and explained using the concept of satisfactoriness. The bi-level multi-objective with multiple interconnected decision makers is discussed in [8]. Three-level programming (TLP) is another class of MLP problems in which there are three independent decision-makers (DMs) [9,10]. Each DM attempts to optimize his objective function and is affected by the actions of the other DMs. Several three-level programming problems such as:

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1. the hybrid extreme-point search algorithm [3,11],
2. mixed-integer problem with complementary slackness [9],
3. the penalty function approach [6,9], and
4. the balance space approach [12–14]

are studied and introduced along with their solution methods.

A bibliography of the related references on bi-level and multi-level programming in both linear and non-linear cases, which is updated biannually, can be found in [15]. The use of the fuzzy set theory [16] for decision problems with several conflicting objectives was first introduced by Zimmermann [17] Thereafter, various versions of fuzzy programming (FP) have been investigated and widely circulated in literature [9,11,18–21].

In a hierarchical decision making context, it has been realized that each DM should have a motivation to cooperate with the other, and a minimum level of satisfaction of the DM at a lower-level must be considered for the overall benefit of the organization. The use of the concept of membership function of fuzzy set theory to multi-level programming problems for satisfactory decisions was first introduced by Lai [22] in 1996. Thereafter, Lai's satisfactory solution concept was extended by Shih et al. [23] and a supervised search procedure with the use of max–min operator of Bellman and Zadeh [24] was proposed. Abo-Sinna [5,10] extended the fuzzy approach for multi-level programming problems of Shih et al. [23] for solving bi-level and three-level non-linear multi-objective programming problems. The basic concept of these fuzzy programming (FP) approaches is the same as implies that each lower level decision maker optimizes his/her objective function, taking a goal or preference of the first level decision makers into consideration. In the decision process, the membership functions of the fuzzy goals for the decision variables of all the decision makers are taken into consideration and an FP problem is solved with a constraint on an overall satisfactory degree of any upper levels. If the proposed solution is not satisfactory to any upper levels, the solution search is continued by redefining the elicited membership functions until a satisfactory solution is reached [19,25].

The main difficulty arises with the FP approach of Shih et al. is that there is possibility of rejecting the solution again and again by the FLDM and re-evaluation of the problem is repeatedly needed to reach the satisfactory decision, where the objectives of the DMs are over conflicting. Even inconsistency between the fuzzy goals of the objectives and the decision variables may arise. This makes the solution process a lengthy one [19,25]. The fuzzy goal programming (FGP) technique introduced by Mohamed [18] – for proper distribution of decision powers to the DMs to arrive at a satisfying decision for the overall benefit of the organization – was developed to overcome the above undesirable situation. The FGP of Mohamed [18] was extended to solve multiobjective linear fractional programming problems in [20], bi-level programming problems in [19], bi-level quadratic programming problems in [25]. In [26], his FGP is further extended to multi-level programming problems with a single objective function at each level.

In this article, the FGP approach introduced by Mohamed [18] is extend to solve multi-level multi-objective linear programming (ML-MOLP) problems. Two FGP procedures are presented in this article to ML-MOLP problems. To formulate any of these two proposed FGP models of the TL-MOLP problem, the fuzzy goals of the objectives are determined by finding individual optimal solutions. They are then characterized by the associated membership functions. These functions are transformed into fuzzy flexible membership goals by means of introducing over and under deviational variables and assigning highest membership value (unity) as aspiration level to each of them. To elicit the membership functions of the decision vectors controlled by any level DM, the optimal solution of the corresponding MOLP problem is separately determined. A relaxation of the decisions are considered to avoid decision deadlock.

The first proposed FGP procedure makes an extension of the work of Pal et al. [19] and Pramanik and Roy [26]. Pal et al. [19] deals with bi-level linear single objective programming problems; and Pramanik and Roy [26] propose an FGP procedure to multi-level programming problems with a single linear objective at each level. The final fuzzy model of Pramanik and Roy groups the membership functions for the defined fuzzy goals of the objective functions and the decision variables at all levels which are evaluated separately for each level except the lower level of the multi-level problem.

The second proposed procedure may be seen as lexicographic methods for solving multiobjective programming problems. Firstly, it formulates the FGP model of the first level problem to obtain a satisfactory solution to the FLDM problem. A relaxation of the FLDM decisions is considered to avoid a decision deadlock. These decisions of the FLDM are modeled by membership functions of fuzzy set theory and passed to the second level DM (SLDM) as additional constraints. Then, the SLDM formulates its FGP model that takes into consideration the membership goals of the objectives and decision variables of the FLDM. Thereafter, the attained solution is sent to the third-level DM (TLDM) who seeks the solution in a similar manner. The process continues until the lower level. This procedure may be considered as extension of the fuzzy mathematical programming algorithm of Shih et al. concept [23] that modified by Sinha in [21,28] following the FGP approach of Mohamed [18].

2. Problem formulation

Consider a p-level programming problem of minimization-type multi-objective functions at each level. Let DM_i denote the decision maker at the ith level that has control over the decision variable $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in_i}) \in R^{n_i}, \quad i = 1, 2, \ldots, p$, where $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, x_p) \in R^n$ and $n = n_1 + n_2 + \cdots + \boldsymbol{n_p}$ and furthermore assume that

$$F_i(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p) \equiv F_i(\boldsymbol{x}) : R^{n_1} \times R^{n_2} \times \cdots \times R^{n_p} \to R^{m_i}, \quad i = 1, 2, \ldots, p \tag{1}$$