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Belief linear programming

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ABSTRACT

This paper proposes solution approaches to the belief linear programming (BLP). The BLP problem is an uncertain linear program where uncertainty is expressed by belief functions. The theory of belief function provides an uncertainty measure that takes into account the ignorance about the occurrence of single states of nature. This is the case of many decision situations as in medical diagnosis, mechanical design optimization and investigation problems. We extend stochastic programming approaches, namely the chance constrained approach and the recourse approach to obtain a certainty equivalent program. A generic solution strategy for the resulting certainty equivalent is presented.

1. Introduction

In stochastic programming, uncertainty on parameters is characterized by a known probability distribution. In many cases, the complete knowledge of the probability is not possible. Incomplete knowledge about probability distribution was considered in many cases. Dupacova [10], for example, studied stochastic programs where the probability distribution is expressed by some of its moments. Recently, Ben Abdelaziz and Masri [3] addressed the problem of stochastic programming with fuzzy probability distribution. In the literature, two main approaches were used to solve stochastic program and stochastic program with incomplete knowledge on probability distributions, namely, the recourse approach and the chance constrained approach. Each approach, under predefined hypotheses, leads to a certainty equivalent mathematical program.

In the case where the decision maker (DM) assigns a probability mass to subsets of the set of all states of nature Θ and not to each individual state of nature $w \in \Theta$, the uncertainty can be modeled as a belief function [21]. A belief function can also be considered when the DM handles incomplete observations, imprecise judgments and/or missing data. Despite the development of many researches using belief function to model uncertainty, attempts to provide decision models under a belief function framework are rather scarce.

The first belief decision models are due to Strat [24] and Jaffray [16]. Inspired from the Hurwicz principle, they proposed a weighted average of the upper and lower expectations related to all probability distributions that have the given belief function as a lower envelope. Yager [26] adapted the ordered weighted averaging (OWA) operators to provide an unifying framework for belief decision making. The Yager's model seems to be difficult to implement as we need to subjectively define the decision maker's (DM) coefficient of optimism and then solve a nonlinear program to define the OWA weights. Denoeux [9] examined some decision strategies for pattern classification in the context of Dempster–Shafer theory. Recently, Boujelben et al. [5] proposed a multiple criteria decision model, inspired by ELECTRE I, where the weights of criteria are expressed by a belief function.

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As probability theory is a special case of evidence theory, few methods were proposed to map a belief function into a probability distribution with some predefined assumptions [14]. Among these methods, Smets [22,23] proposed the transferable belief model (TBM) that allows the DM to transform any belief decision model into a probabilistic decision problem by converting belief functions into probabilities using the pignistic transformation.

In the literature, the first attempt to incorporate belief functions within optimization problems was related to the reliability based design optimization (RBDO) problem [1]. The resulting program is called the evidence-based design optimization (EBDO) problem where the plausibility of the performance constraint violation has to be small [17]. Mourelatos and Zhou [18] proposed a hybrid solution algorithm for the EBDO problem where they apply first an RBDO algorithm to move to the vicinity of the optimum and then they use a derivative free optimizer that considers only the obtained active constraints to find the evidence-based optimum solution. Recently, Hermann [12] proposed an unified solution approach for both the EBDO and the imprecise probability design optimization (IPDO) problem.

In this paper, we introduce a general uncertain linear programming where uncertainty is characterized by a belief function. The resulting optimization program is called the belief linear program (BLP). As probability distributions are belief functions, we propose to generalize main solution approaches in stochastic programming, namely the chance constrained approach and the recourse approach, to state for a generic solution approaches.

In the next section, we recall some basic concepts of the belief function theory and then in Section 3, we introduce the BLP problem. In Section 4, we illustrate with an example the way we generate certainty equivalents to stochastic programs under the hypotheses of the chance constrained approach and the recourse approach. In Section 5, we extend the chance constrained approach, that we call belief constrained approach, to get a certainty equivalent program to the BLP problem. We discuss the convexity of the obtained certainty equivalent and we propose a solution strategy that can be used to solve it. In Section 6, we present the recourse approach for the BLP problem and a solution algorithm to solve the resulting certainty equivalent. All concepts introduced through the paper are illustrated with a simple example.

2. Basic concepts of the belief function theory

The belief function theory, also called the Dempster–Shafer theory of evidence, was initiated by Dempster [8] and then extended by Shafer [21]. Compared to other uncertainty measures such us fuzzy sets theory, the belief function theory is an extension to the probabilistic reasoning. The belief function is obtained using a probability distribution over the power set of the set of possible events. This allows assigning a mass of evidence (or probability of occurrence) to subsets and not only to singletons. In many decision situations, we can only measure the occurrence of a set of events, for example, in the medical diagnosis problems; we might have evidences about the presence of a set of bacteria and no evidence about the presence of particular bacteria [11].

Let us denote by $\Theta = \{w_1, \ldots, w_N\}$ the set of mutually exclusive states of nature and 2^{Θ} the power set of Θ, called the frame of discernment. Over this frame, we define a probability distribution m

$$m : 2^{\Theta} \to [0, 1],$$

such that

$$m(\phi) = 0 \quad \text{and} \quad \sum_{A \subseteq \Theta} m(A) = 1.$$

This function is called basic probability assignment (bpa). Obviously, m divides one unity over singletons and subsets of the frame of discernment. Each singleton or each subset with a nonzero mass is called focal element. The basic probability assignment function is a generalization of the probability mass function in probability theory where focal elements are not only singleton [21].

Associated with the bpa, we introduce two measures called belief measure (Bel) and plausibility measure (Pl) and are respectively defined by:

$$\begin{aligned} Bel: \quad & 2^{\Theta} \to [0, 1], \\ & A \to Bel(A) = \sum_{B \subseteq A} m(B), \end{aligned}$$

and

$$\begin{aligned} Pl: \quad & 2^{\Theta} \to [0, 1], \\ & A \to Pl(A) = \sum_{A \cap B \neq \phi} m(B). \end{aligned}$$

For all sets $A \subseteq \Theta$, $Bel(A)$ is the total mass of evidence attributed by m to the subsets of A and $Pl(A)$ is the maximum degree of evidence that can be assigned to A [21].