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A new method for solving fully fuzzy linear programming problems

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ABSTRACT

Lotfi et al. [Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution, Appl. Math. Modell. 33 (2009) 3151–3156] pointed out that there is no method in literature for finding the fuzzy optimal solution of fully fuzzy linear programming (FFLP) problems and proposed a new method to find the fuzzy optimal solution of FFLP problems with equality constraints. In this paper, a new method is proposed to find the fuzzy optimal solution of same type of fuzzy linear programming problems. It is easy to apply the proposed method compare to the existing method for solving the FFLP problems with equality constraints occurring in real life situations. To illustrate the proposed method numerical examples are solved and the obtained results are discussed.

1. Introduction

Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment. Many researchers adopted this concept for solving fuzzy linear programming problems [2–8]. However, in all of the above mentioned works, those cases of fuzzy linear programming have been studied in which not all parts of the problem were assumed to be fuzzy, e.g., only the right hand side or the objective function coefficients were fuzzy but the variables were not fuzzy.

The fuzzy linear programming problems in which all the parameters as well as the variables are represented by fuzzy numbers is known as FFLP problems. FFLP problems can be divided into two categories: (1) FFLP problems with inequality constraints (2) FFLP problems with equality constraints. Some authors [9–11] have proposed different methods for solving FFLP problems with inequality constraints. In all these methods firstly the FFLP problem is converted into crisp linear programming problem and then the obtained crisp linear programming problem is solved to find the fuzzy optimal solution of the FFLP problems. The main disadvantage of the solution, obtained by the existing methods, are that it does not satisfies the constraints exactly i.e. it is not possible to obtain the fuzzy number of the right hand side of the constraint by putting the obtained solution in the left hand side of the constraint.

Dehghan et al. [12] proposed a fuzzy linear programming approach for finding the exact solution of fully fuzzy linear system (FFLS) of equations. Lotfi et al. [13] proposed a method to obtain the approximate solution of FFLP problems. To the best of our knowledge, till now there is no method in the literature to obtain the exact solution of FFLP problems with equality constraints. By using the existing method [13] the obtained solutions are approximate not exact and also it is very difficult to apply the existing method [13] to find the fuzzy optimal solution of FFLP problems.

In this paper the shortcomings of the existing methods [12,13] are pointed out and to overcome these shortcomings, a new method is proposed for finding the fuzzy optimal solution of FFLP problems with equality constraints. To illustrate the proposed method, numerical examples are solved and the obtained results are discussed.

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This paper is organized as follows: In Section 2 some basic definitions and arithmetics between two triangular fuzzy numbers are reviewed. In Section 3 formulation of FFLP problems and application of ranking function for solving FFLP problems are discussed. In Section 4 shortcomings of the existing methods are pointed out. In Section 5 a new method is proposed for solving FFLP problems. To illustrate the proposed method, numerical examples are solved and the obtained results are discussed in Section 6. In Section 7 advantage of the proposed over the existing method are described. Conclusions are discussed in Section 8.

2. Preliminaries

In this section, some necessary backgrounds and notions of fuzzy set theory are reviewed.

2.1. Basic definitions

Definition 2.1 [14]. The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 [14]. A fuzzy number $\tilde{A} = (a,b,c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leqslant x \leqslant b, \\ \frac{(x-c)}{(b-c)}, & b \leqslant x \leqslant c, \\ 0, & \text{otherwise}. \end{cases}$$

Definition 2.3 [14]. A triangular fuzzy number (a,b,c) is said to be non-negative fuzzy number iff $a \geqslant 0$.

Definition 2.4 [14]. Two triangular fuzzy numbers $\tilde{A} = (a,b,c)$ and $\tilde{B} = (e,f,g)$ are said to be equal if and only if $a = e$, $b = f$, $c = g$.

Definition 2.5 [15]. A ranking function is a function $\Re : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (a,b,c)$ be a triangular fuzzy number then $\Re(\tilde{A}) = \frac{a+2b+c}{4}$.

2.2. Arithmetic operations

In this subsection, arithmetic operations between two triangular fuzzy numbers, defined on universal set of real numbers R, are reviewed [14].

Let $\tilde{A} = (a,b,c)$ and $\tilde{B} = (e,f,g)$ be two triangular fuzzy numbers then

(i) $\tilde{A} \oplus \tilde{B} = (a,b,c) \oplus (e,f,g) = (a+e, b+f, c+g)$,
(ii) $-\tilde{A} = -(a,b,c) = (-c,-b,-a)$,
(iii) $\tilde{A} \ominus \tilde{B} = (a,b,c) \ominus (e,f,g) = (a-g, b-f, c-e)$,
(iv) Let $\tilde{A} = (a,b,c)$ be any triangular fuzzy number and $\tilde{B} = (x,y,z)$ be a non-negative triangular fuzzy number then

$$\tilde{A} \otimes \tilde{B} \simeq \begin{cases} (ax, by, cz), & a \geqslant 0, \\ (az, by, cz), & a < 0,\ c \geqslant 0, \\ (az, by, cx), & c < 0. \end{cases}$$

3. Fully fuzzy linear programming problem

Linear programming is one of the most frequently applied operations research technique. In the conventional approach value of the parameters of linear programming models must be well defined and precise. However, in real world environment, this is not a realistic assumption. In the real life problems there may exists uncertainty about the parameters. In such