Mathematical and Computer Modelling 53 (2011) 1878–1888

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Sensitivity analysis in fuzzy number linear programming problems

A. Ebrahimnejad

Islamic Azad University, Qaemshahr Branch, Department of Mathematics, Qaemshahr, Iran

ARTICLE INFO

Article history:
Received 2 May 2009
Received in revised form 6 January 2011
Accepted 6 January 2011

Keywords:
Fuzzy number linear programming
Fuzzy primal and dual simplex algorithms
Ranking function
Trapezoidal fuzzy numbers

ABSTRACT

In this paper, we generalize the concept of sensitivity analysis in fuzzy number linear programming (FLNP) problems by applying fuzzy simplex algorithms and using the general linear ranking functions on fuzzy numbers. The purpose of sensitivity analysis is to determine changes in the optimal solution of FNLP problem resulting from changes in the data. If the change affects the optimality of the basis, we perform primal pivots to achieve optimality by use of the fuzzy primal simplex method. Whenever the change destroys the feasibility of the optimal basis, we perform dual pivots to achieve feasibility by use of the fuzzy dual simplex method.

1. Introduction

The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al. [1] in the framework of the fuzzy decision of Bellman and Zadeh [2]. The first formulation of fuzzy linear programming (FLP) is proposed by Zimmermann [3]. Afterwards, many authors have considered various kinds of FLP problems and have proposed several approaches for solving these problems [4–14]. To solve a multi objective programming problem with fuzzy coefficients Wu [15] transformed the problem into a vector optimization problem by applying the embedding theorem and a suitable linear defuzzification function. Katagiri et al. [16] considered a multi-objective 0–1 programming problem with fuzzy random variables as coefficients of objective functions and proposed a decision-making model maximizing the expected degrees of possibility that the objective function values attained the fuzzy goals. Iskander [17] introduced a stochastic fuzzy linear multi objective programming problem and transformed it to a stochastic fuzzy linear programming problem using a fuzzy weighted objective function. They, used chance-constrained approach as well as the α-level methodology to transform the stochastic fuzzy linear programming problem to its equivalent deterministic-crisp linear programming problem.

In addition, some authors have used the concept of comparison of fuzzy numbers to solve fuzzy linear programming problems. In fact, most convenient methods are based on the concept of comparison of fuzzy numbers by using linear ranking functions [5,18,9–12]. Of course, linear ranking functions have been proposed by researchers to suit their requirements of the problem under consideration and conceivably there are no generally accepted criteria for application of ranking functions. Nevertheless, usually in such situations authors define a crisp model which is equivalent to an FLP problem and then use optimal solution of the model as the optimal solution of the FLP problem. Based on this idea Maleki et al. [12] proposed a new method for solving the FNLP problem and used its solution to obtain the fuzzy solution of the fuzzy variable linear programming (FVLP) problem. Mahdavi-Amiri and Nasseri [9] extended the concepts of duality in FNLP problems as a similar problem leading to the dual simplex algorithm [19] for solving such problems. However, the method of Malkei et al. [12] has a shortcoming and it is not efficient when the decision variables are bounded in the FNLP problem. Thus, some authors proposed a new approach to overcome this shortcoming based on dual and primal simplex methods [7,8]. Moreover, Mahdavi-Amiri and Nasseri [10] used a certain linear ranking function to define the dual of FNLP problems

E-mail addresses: a.ebrahimnejad@srbiau.ac.ir, aemarzoun@gmail.com.

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.01.013

as FVLP problems that lead to an efficient method called the dual simplex algorithm for solving FVLP problems directly. After that, Ebrahimnejad et al. [5] gave another efficient method namely primal–dual simplex algorithm to obtain a fuzzy solution of FVLP problems. Also, Nasseri and Ebrahimnejad [13] applied a fuzzy primal simplex method [11] to solve flexible linear programming problems directly without solving any auxiliary problem. Moreover, Ebrahimnejad and Nasseri [4] used the complementary slackness to solve FNLP and FVLP problems without the need of a simplex table. Hosseinzadeh Lotfi et al. [20] discussed full FLP problems of which all parameters and variable triangular fuzzy numbers. They used the concept of the symmetric triangular fuzzy number and introduced an approach to defuzzify a general fuzzy quantity. Ganesan and Veeramani [21] introduced a new method for solving a kind of linear programming problem involving symmetric trapezoidal fuzzy numbers without converting them to crisp linear programming problems based on primal simplex algorithm. Ebrahimnejad et al. [6] developed their method for situations in which some or all fuzzy decision variables are bounded. Some authors [14,22] developed the dual of a linear programming problem with symmetric trapezoidal fuzzy numbers without converting them to crisp linear programming problems.

In published works on fuzzy linear programming there are only a few papers dealing with stability or sensitivity analysis in fuzzy mathematical programming. Sensitivity analysis in FLP problems (with crisp parameters and soft constraints) was first considered by Hamacher et al. [23], where a functional relationship between changes of parameters of the right-hand side and those of the optimal value of the primal objective function was derived for almost all conceivable cases. Sensitivity analysis for fuzzy linear fractional programming problems (FLFP) was studied by Dutta et al. [24]. Tanaka et al. [25] have discussed the value of information in an FLP problem (with symmetrical triangular fuzzy numbers) via sensitivity analysis. Fullér [26]investigated the stability of the solution of FLP problems with respect to changes of centers of fuzzy parameters. He showed that the solution to these problems is stable under variations in the membership function of the fuzzy coefficients. In this paper, we generalize the concept of sensitivity analysis on the parameters of the crisp linear programming [27] to the fuzzy number linear programming and show that the fuzzy primal simplex algorithm stated in [11] and the fuzzy dual simplex algorithm presented in [19] would be useful for post optimality analysis on linear programming problems with fuzzy numbers.

The rest of this paper is organized as follows: In Section 2, we give some necessary concepts of fuzzy set theory as well as a review of the fuzzy number linear programming problem and it's dual problem. We review the fuzzy simplex algorithms to solve the fuzzy number linear programming problems in Section 3 and then explain them by several illustrative examples. The fuzzy simplex algorithms are used for sensitivity analysis in Section 4. Finally, we conclude in Section 5.

2. Preliminaries

2.1. Fuzzy arithmetic operators and ranking

In this section, we give some basic concepts and results of fuzzy numbers, fuzzy arithmetic and ranking of fuzzy numbers which are needed in the rest of the paper (taken from [10]).

A fuzzy set is defined as a subset $\tilde{a}$ of universal set $X \subseteq \mathrm{R}$ by its membership function $\mu_{\tilde{a}}(.)$, which assigns to each element $x \in \mathrm{R}$, a real number $\mu_{\tilde{a}}(x)$ in the interval [0, 1]. The α-cut or α-level of a fuzzy set $\tilde{a}$, which plays an essential role in fuzzy optimization, is defined as an ordinary set $[\tilde{a}]_\alpha$ for which the degree of its membership function exceeds the level α. A fuzzy number is a convex normalized fuzzy set of the real line R; whose membership function is piecewise continuous. The set of fuzzy numbers on R is denoted by F(R).

Definition 2.1. A fuzzy number $\tilde{a} = (a_1, a_2, \alpha_1, \alpha_2)$ is said to be a trapezoidal fuzzy number, if its membership function is given by function:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-(a_1-\alpha_1)}{\alpha_1} & \text{for } a_1-\alpha_1 \le x \le a_1 \\ 1 & \text{for } a_1 \le x \le a_2 \\ \dfrac{(a_2+\alpha_2)-x}{\alpha_2} & \text{for } a_2 \le x \le a_2+\alpha_2 \\ 0 & \text{else.} \end{cases} \tag{1}$$

Now, we define arithmetic on trapezoidal fuzzy numbers. Let $\tilde{a} = (a_1, a_2, \alpha_1, \alpha_2)$ and $\tilde{b} = (b_1, b_2, \beta_1, \beta_2)$ be two trapezoidal fuzzy numbers. Define,

$$\begin{aligned} &x > 0,\ x \in \mathrm{R}; \quad x\tilde{a} = (xa_1, xa_2, x\alpha_1, x\alpha_2), \\ &x < 0,\ x \in \mathrm{R}; \quad x\tilde{a} = (xa_2, xa_1, -x\alpha_2, -x\alpha_1), \\ &\tilde{a} + \tilde{b} = (a_1+b_1, a_2+b_2, \alpha_1+\beta_1, \alpha_2+\beta_2). \end{aligned}$$

One convenient approach for solving fuzzy linear programming problems is based on the concept of comparison of fuzzy numbers by use of ranking functions (see [10]). An effective approach for ordering the elements of F(R) is to define a ranking