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# Optimization of tower crane and material supply locations in a high-rise building site by mixed-integer linear programming

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ABSTRACT

Facility layout design and planning within construction sites are a common construction management problem and regarded as a complex combinatorial problem. To transport heavy materials, tower cranes are needed and should be well located to reduce operating costs and improve overall efficiency. Quadratic assignment problem (QAP), non-linear in nature, has been developed to simulate the material transportation procedure. Applying linear constraint sets, the quadratic problem can be linearized and the problem could be formulated into a mixed-integer-linear programming (MILP) problem solvable by a standard branch-and-bound technique for true optimal results. Numerical findings show that MILP results outperform those optimized by Genetic Algorithms with almost 7% on improving the objective function values in which facilities and locations can be modeled using integer variables. To demonstrate the design flexibility of using MILP formulation, the problem is also extended to non-homogeneous storages where different materials can be stored at a single supply point.



## 1. Introduction

Tower cranes are common at construction sites nowadays which are often erected hundreds of feet in height to lift heavy and bulky objects like steel beams, ready mixed concrete, prefabricated elements, large tools such as machinery and equipment, and a wide variety of other building materials within a construction site. They are usually located at a convenient and safe place where most of these heavy and bulky materials can be handled. Ideally, their jib should reach and cover any part of the buildings in a construction site to lift and drop construction materials over various supply and demand points. It is expected that there are many factors to be considered while locating a tower crane to undertake heavy material transportation tasks efficiently in terms of operating costs and transportation distances.

Having a good facility layout including the tower crane and material supply locations is one of the most important parts to increase such production efficiency in construction sites, especially in most metropolitan cities like Hong Kong where the sites are usually confined in nature with a limited area due to the scarcity of land supply. To cope with such construction site conditions, practitioners in the industry relying much on experiences always lack a well-defined approach to come up with an optimal site layout for construction projects [12,28].

Nowadays, building construction projects are highly mechanized employing tower cranes to transport heavy construction materials [24]. As material transportation is one of the major activities in the building construction industry, lifting and hoisting heavy materials by cranes in construction sites are common tasks that require meticulous planning [21]. Construction cranes are classified into tower cranes and mobile cranes. Tower crane, which is suitable for a wide range of work assignments and site conditions, is one of the key facilities for vertical and horizontal transportation of materials, especially for the heavy prefabrication units and large panel formwork in high-rise construction [28]. The demand for tower cranes is rising according to American contractors [24], and crane rental companies have increased the proportion of tower cranes in their equipment fleets [4,5,26]. Locations of tower cranes and surrounding material supply points are critical to the overall efficiency in a construction site. The objective of this paper is to formulate the design problem for a construction site layout involving locating a single tower crane and associated material supply points into a mixed-integer linear program to minimize the total operating cost.

## 2. Literature review

Two traditional models, Quadratic Assignment Problem (QAP) and Graph-Theoretic, have been developed to mathematically simulate the procedure of material distribution in facility layout problems originally designed for the manufacturing industry [19]. The former one is frequently implemented in layout planning for the construction industry in recent years. It is classified as a difficult problem in the NP-

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hard class that was introduced to design plant locations [6,14]. As the layout efficiency is generally evaluated by the total material handling cost, the total interplant transportation cost could be set as the objective function for optimization [15]. In their formulation, the interdepartmental flows prescribed the material flow from one department to another department, the unit transportation cost specified the cost to move one unit load in one unit distance between any two departments, the transportation distance measured the rectilinear distance from the centroids of two locations where the departments are assigned. Mathematically, facilities' locations can be represented by a permutation matrix containing a set of binary-type integer variables with each row sum and each column sum being "1" to ensure a one-to-one mapping relationship. Each entry of the matrix represents the assigned location for the corresponding facility. The quadratic assignment problem (QAP) could be formulated as a problem of minimizing the objective function with respect to the permutation variables. Its nature is quadratic because there is a product term of two binary variables in the formulation [27].

It is well-known that QAP is one of the non-linear optimization problems and relevant optimization results would become sub-optimal [9]. Generally, there are two approaches to solve combinatorial problems—local improvement method and global improvement method. The strategy in local improvement method is to repeat searching the neighborhood of predefined permutation until no further improvement could be found. Genetic algorithms (GAs), which is one of the most popular heuristic methods, apply probabilistic search logic [13] that operates well in all kinds of objective functions and even non-linear solution space. Applying genetic algorithms to optimize the material storage locations in a building construction can be found in Adel El-Baz [1], Azadivar and Wang [3], Kaku et al. [14], Matsuzaki et al. [18] and Fung et al. [12]. Genetic algorithms have also been applied to allocate construction facilities [7] and optimize facility layouts [17] in construction sites. The challenge, however, remains in finding an appropriate problem representation that results in an efficient and successful implementation of the algorithm [32,35].

The QAP has attracted interests not only for its wide applicability, but also because it permits rich variety of ways to get relaxations which will lead to a global optimized solution in a limited time [9]. The relaxation approaches include Linear Relaxations [16], Orthogonal Relaxation [30], Semidefinite Relaxations [34] and Convex Quadratic Relaxation [2]. One of the earliest and least expensive relaxations is the Linear Relaxation which is known as the Gilmore–Lawler bound (GLB). Lawler [16] suggested using a single term in the form of a binary variable to replace the product term in resolving the non-linearity in the formulation.

By replacing the quadratic terms in the QAP formulation with simplified linear terms, the formulation could be turned into a mixed-integer linear programming (MILP) formulation. Generally, linearizing a QAP into a MILP will expand the problem by inducing a huge number of variables and constraints [11]. It should be well noted that linearizations and constraint relaxations to form a MILP solved by standard procedures such as the branch-and-bound technique could reach a global optimum solution [31]. Montreuil [20] first applied the MILP formulation for the facility layout and material handling problems. Easa and Hossain [10] studied the facility allocation problem adding visual and shape constraints in a continuous solution space for optimization using MILP approach.

There are many research works about the location and transportation time of a tower crane that have been proposed: Choi and Harris [8] raised a mathematical model for determining the most suitable tower crane location; Zhang et al. [33] used the Monte Carlo simulation approach to optimize tower crane location; Tam and Tong [29] developed an artificial neural network model for predicting tower crane operations and genetic algorithm model for site facility layout [28,29]. The problem nature of locating a tower crane and material supply locations in a building construction site is very similar to a conventional facility location problem except that a 3-D (dimensional) consideration for the material transportation through the hook movements of a tower crane must be provided. We will then formulate the tower crane location problem into a binary-mixed-integer-linear program and apply a standard branch-and-bound technique to look for the global optimum solution.

## 3. Optimization of tower crane location and material supply points using mixed-integer linear programming

*List of symbols*

| | |
|---|---|
| $x$ | Position at $x$-axis |
| $y$ | Position at $y$-axis |
| $z$ | Position at $z$-axis |
| $Cr_k^x, Cr_k^y, Cr_k^z$ | Coordinate of a tower crane at location $k$; |
| $D_j^x, D_j^y, D_j^z$ | Coordinate of a demand point at location $j$; |
| $S_i^x, S_i^y, S_i^z$ | Coordinate of supply point at location $i$; |
| $V_h$ | Hoisting velocity of hook (m/min); |
| $V_\omega$ | Slewing velocity of jib(r/min); |
| $V_r$ | Radial velocity (m/min); |
| $T_{r(i,j)}^k$ | Time for trolley radial movement of a tower crane at location $k$ from a supply point $i$ to a demand point $j$; |
| $T_{\omega(i,j)}^k$ | Time for trolley tangent movement of tower crane at location $k$ from a supply point $i$ to a demand point $j$; |
| $T_{h(i,j)}^k$ | Time for hook horizontal movement of tower crane at location $k$ from a supply point $i$ to a demand point $j$; |
| $T_{v(i,j)}$ | Time for hook vertical movement from a supply point $i$ to a demand point $j$; |
| $T_{i,j}^k$ | Hook total travel time of tower crane at location $k$ between a supply point $i$ and a demand point$j$; |
| $\alpha$ | Degree of coordination of hook movement in radial and tangential directions in horizontal plane ranging between 0.0 and 1.0 continuously (where 0 stands for full simultaneous movement and 1 for full consecutive movement); |
| $\beta$ | Degree of coordination of hook movement in vertical and horizontal planes ranging between 0.0 and 1.0 continuously (where 0 stands for full simultaneous movement and 1 for full consecutive moment); |
| $\gamma_k$ | Degree of difficulty in hook movement control for the tower crane at location $k$ ranging between 0.1 and 10.0 continuously (where 1.0 represents operation under normal condition, 0.1 in a fast operating mode with advance video system support and 10.0 in a very slow operation due to difficult site condition); |
| $i$ | Available material supply points; |
| $I$ | Total number of supply points to be allocated within the site area; |
| $j$ | Potential material demand points; |
| $J$ | Total number of demand points in the site; |
| $k$ | Available tower crane location; |
| $K$ | Total number of available tower crane locations; |
| $l$ | Material type; |
| $L$ | Total number of material types; |
| $Q$ | Total quantity of all the materials flowing between supply point '$i$' and demand point '$j$' per concrete floor cycle; |
| $Q_{l,j}$ | Quantity of the material type '$l$' flowing at demand point '$j$'; |
| $\Delta_j$ | Demand point '$j$' is selected where '1' is yes but '0' is no; |
| $\zeta_k$ | Tower crane at location '$k$' is selected where '1' is yes but '0' is no; |
| $\chi_{i,l}$ | Decision of assigning material '$l$' to supply point '$i$' where '1' is yes but '0' is no; |
| $y_{i,j}$ | Decision of connecting supply point '$i$' with demand point '$j$' where '1' is yes but '0' is no; |
| $\delta_{i,j,k,l}$ | An auxiliary binary-type variable where '1' means material '$l$' is transferred by a tower crane at location '$k$' from a supply point '$i$' to a demand point '$j$' but '0' otherwise; |