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An inexact rough-interval fuzzy linear programming method for generating conjunctive water-allocation strategies to agricultural irrigation systems

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ABSTRACT

An inexact rough-interval fuzzy linear programming (IRFLP) method is developed for agricultural irrigation systems to generate conjunctive water allocation strategies. The concept of "rough interval" is introduced in the modeling framework to represent dual-uncertain parameters. After the modeling formulation, an agricultural water allocation management system is provided to demonstrate the applicability of the developed method. The results show that reasonable solutions and allocation strategies are obtained. Based on the analysis of alternatives obtained from different scenarios, the significant impact of dual uncertainties existing in the system is specified. Comparisons between the results from IRFLP and interval-valued fuzzy linear programming are also conducted. The obtained rough-interval solutions correspond to the management strategies under both normal and special system conditions, and thus more conveniences would be provided for decision makers. Compared to the previous modeling efforts, the proposed IRFLP shows uniqueness in addressing the interaction between dual intervals of highly uncertain parameters, as well as their joint impact on the system.

1. Introduction

The lack of efficient, equitable and sustainable water management and effective policy instructions from decision makers becomes more and more challenging across the world [1–4]. This challenge is particularly intense in agricultural irrigation systems, where scarcity of fresh water resources in many arid and semi-arid regions keeps growing. A sustainable water management policy for agricultural irrigation is to promote water use in such a way that society's needs are met to the extent possible now and in the future [5]. In review of this necessity, a number of research efforts have been undertaken to facilitate decision makers regulating sustainable water resources management policies [6–11]. Considerable attention has been given to the integrated use of surface and ground water resources [12]. For example, Maknoon and Burges [13] listed out the chronological development of a conjunctive use approach from its recognition. Coe [14] firstly postulated the definition and types of conjunctive use projects. Mohan and Jothiprakash [15] developed a combined optimization-simulation approach for evaluating the alternate priority-based policies for operation of surface and ground water and applied it to a case study located in India for demonstrating its applicability.

In the previous studies, linear programming approaches were always used to obtain optimal strategies, such as water-allocation patterns, crop-planting plans, and canal-expansion schemes, with an objective of maximizing net benefit

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[16,17]. Stewart et al. [18] developed a model to specify the irrigation schedule for three crops in a stream-aquifer irrigation system. It was then used for conditions of limited and unlimited water availability. Sritharan et al. [19] studied an on-farm design and water management planning through a two-stage programming approach. Kumar and Pathak [20] presented a linear programming model for optimal crop planning in a canal-aquifer system. Sensitivity analysis was made on available crop area, canal water supply, groundwater extraction, and operation and maintenance costs of the system. Paudyal and Gupta [21] studied irrigation management by multilevel optimization. Abrishamchi et al. [22] carried out reservoir planning for irrigation districts by using a chance-constrained optimization model. Maqsood et al. [23] developed an interval-parameter two-stage optimization model (ITOM) for water resources planning in an agricultural system under uncertainty. A variety of decision alternatives were generated and post-optimality analysis was also performed under combinations of water shortages through a factorial design approach.

However, the existence of complex uncertainties makes it difficult in obtaining optimal strategies with high feasibility and robustness [24,25]. In practices, dual and even multiple uncertainties may exist particularly in those systems with high vagueness and ambiguity. In agricultural irrigation systems, for example, when the quantity of an unregulated water source is highly imprecise, it has to be estimated in terms of a decision maker's subjective judgment and objective monitoring data. Based on the investigation, the decision maker identified that the water quantity ranges from 12 to 18 $\times 10^6$ m^3 per year under most cases; however, in several special years, the range may change to 10 to 20 $\times 10^6$ m^3. For this type of information, conventional uncertainty analysis methods can neither reflect its dual-layer feature, nor entirely deliver it to the resulting decision schemes. In response to this concern, a number of programming methods that are capable of dealing with dual and multiple uncertainties have been recently proposed [7,16,26–30]. These efforts can deal with dual and multiple uncertainties existing in both modeling parameters (e.g. dual-interval parameters, interval-valued fuzzy sets, and fuzzy boundary intervals) and relationships (e.g. inexact fuzzy programming, inexact stochastic programming, and fuzzy-stochastic programming).

Among these efforts, the fuzzy sets theory was particularly considered in terms of its capability in tackling subjective information derived from decision makers. Special attention can be paid to an interval-valued fuzzy linear programming (IVFLP) method where modeling parameters with high vagueness were represented by a type of advanced fuzzy sets with the membership grade of each possible value falling within a range [26–29]. Interval-valued fuzzy sets can simultaneously represent dual-uncertain information arising from subjective experience of decision makers and objective historical data; however, the existing solution method for IVFLP cannot deliver these uncertainties to the corresponding optimal solutions. Thus, development of a method that can reflect dual-uncertain characteristics in both modeling formulation and resulting solutions is necessary. In this study, an inexact rough-interval fuzzy linear programming (IRFLP) method will be developed facing to the above challenge. The concept of "rough interval" will be introduced to represent dual-uncertain information of many parameters, and the associated solution method will be presented to solve IRFLP problems providing dual-uncertain solutions. After the modeling formulation, a case study will be provided for demonstrating its applicability and advantages upon the IVFLP approach.

2. Inexact rough-interval fuzzy linear programming

2.1. Inexact interval-valued fuzzy linear programming

Generally, a conventional fuzzy linear programming model can be formulated as follows:

$$\max \quad f = C \cdot X \tag{1a}$$

$$\text{subject to } AX \underset{\sim}{\leqslant} B \tag{1b}$$

$$X \geqslant 0 \tag{1c}$$

where $A \in R^{m\times n}$, $B \in R^{m\times 1}$, $C \in R^{1\times n}$, and $X \in R^{n\times 1}$ (R denotes a set of real numbers), and $\leqslant$ is a fuzzy $\leqslant$ symbol. To enhance the ability of this model in handling uncertain information of parameters in various forms, ĩntervals, dual intervals, fuzzy boundary intervals, and interval-valued fuzzy sets were employed in many research efforts [26,27,30]. These attempts provided potential of using enhanced intervals (instead of deterministic numbers) to express many complex parameters with single, dual and even multiple uncertainties. According to [28,29], an inexact interval-valued fuzzy linear programming (IVFLP) problem can be formulated as follows:

$$\max \quad f^{\pm} = C^{\pm} \cdot X^{\pm} \tag{2a}$$

$$\text{subject to } \widetilde{A}^{\pm} X^{\pm} \leqslant \widetilde{B}^{\pm} \tag{2b}$$

$$X^{\pm} \geqslant 0 \tag{2c}$$

where $\widetilde{A}^{\pm} \in \{\widetilde{R}^{\pm}\}^{m\times n}$; $\widetilde{B}^{\pm} \in \{\widetilde{Q}^{\pm}\}^{n}$; $\widetilde{R}^{\pm}$ and $\widetilde{Q}^{\pm}$ denote fuzzy sets with interval-valued fuzzy membership functions; $C^{\pm} \in \{\Pi\}^{p}$ and Π denotes a set of conventional interval parameters; superscript "±" indicates the corresponding parameters/decision variables show interval feature; $X^{\pm}$ denotes a set of decision variables; $f^{\pm}$ denotes objective function value.