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Compromise Fuzzy Multi-Objective Linear Programming (CFMOLP) heuristic for product-mix determination ☆

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ABSTRACT

This paper models a crisp Linear Programming (LP) as a Compromise Fuzzy Multi-Objective LP (CFMOLP). The application of CFMOLP has been focused on an industrial engineering problem that seeks profit maximisation by optimising product-mix. Imprecision of the large volume of industrial data and the conglomerated decision from all levels of management lead fuzzication of the identified constraints and the objective functions as well. The crisp model described is in the form of crisp-Multi-Objective Linear Programming (MOLP) with objective functions, functional constraints and non-negativity constraints. This model is formulated as a fuzzy-MOLP and subsequently converted into an equivalent compromise-MOLP model. The paper describes the development process for the CFMOLP model and its application along with appropriate interpretation.

1. Introduction and background

Many problems in economics, operations research, decision sciences, engineering and management sciences have mainly been studied from the optimisation point of view. As the decision-making is influenced by the disturbances of social and economical circumstances, straight forward optimisation approach is not always the best. It is because under such influences, many problems are ill-structured. In real-world situations, as reaching to the ideal solution is practically unattainable, a decision-maker considers best feasible solutions closest to the ideal solution instead of ideal solution (Zeleny, 1982). Under this situation, a satisfaction approach is much better than an optimisation one.

Literature reveal variants of Multi-Objective Linear Programming (MOLP) models and their use in decision-making. For example, Karsak and Kuzgunkaya (2002) propose a fuzzy MOLP approach as an alternative to the classical mathematical programming formulation. Their proposal uses triangular fuzzy numbers and does not consider the compromise approach during evaluation of candidate-alternatives. Further, Gao and Tang (2003) propose a MOLP model for purchasing of raw materials of a large-scale steel plant. 'Point estimate weight-sums method' has been used in their work to solve the set of equations. The method converts the MOLP into a general LP problem and the solution is obtained by assigning positive weights only. Their method does not embed a fuzzy technique so as to deal with vagueness of the problem. Further, the efficacy of MOLP has been justified by Downing and Ringuest (1998). They use Excel® and Visual Basic® to implement four different algorithms for MOLP. It has been demonstrated that explicit and effecting modelling of any decision-making process with MOLP algorithms improves the effectiveness of the processes. Interactive "fuzzy linear programming" (FLP) and "fuzzy MOLP" methods have been proposed by Liang (2008, 2006) for solving transportation planning problems considering fuzzy goals, available supply and forecast demand. A mathematical model for the preference-ranking to fuzzy goal of constraints is proposed by Hasuike and Ishii (2009) that considers randomness, fuzziness and flexibility in modelling product-mix decision-support. Tan (2005) proposes the use of symmetric FLP for determining an optimal product-mix solution with multiple objectives is reported. Mathematical models, including a LP model, are proposed by Letmathe and Balakrishnan (2005) in order to estimate an optimal product-mix in presence of multiple constraints. Karakas, Koyuncu, Erol, and Kokangul (2010) report a mathematical model using activity-based costing to determine the optimal product-mix by maximising profit considering fuzziness in demand of the products.

The application arena of the proposed CFMOLP model is "product-mix". Product-mix determination using various models

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Nomenclature

i decision variable index, $i = 1, 2, \ldots, 8$
δ objective function index, $\delta = u, v, w, y, z$
Δ index for the row vector of objective functions, $\Delta = U, V, W, Y, Z$
j 1, 2, 3
u the first objective function which maximizes the revenue
v the second objective function which maximizes the market share for chocolate bars
w the third objective function which maximizes units produced
y the fourth objective function which maximizes plant utilization
z the fifth objective function which maximizes the profit
c_i the objective function central coefficient (i.e., the objective coefficient in the crisp LP model) of the ith decision variable
c_i^- the lower bound of c_i
c_i^+ the upper bound of c_i
$\mu_{c_i}(x; c_i^-, c_i, c_i^+)$ the degree of membership function which gives the degree of membership of an element x to the set of value represented by the term "about the central coefficient c_i" with the lower bound c_i^- and the upper bound c_i^+
$\mathbf{c}_\delta = (c_1 \ldots c_i \ldots c_8)$ the vector of the central coefficients of x_i in the objective function δ
$\mathbf{c}_\delta^- = (c_1^- \ \ldots \ c_i^- \ \ldots \ c_8^-)$ the vector of the lower bound of the coefficients of x_i in the objective function δ
$\mathbf{c}_\delta^+ = (c_1^+ \ \ldots \ c_i^+ \ \ldots \ c_8^+)$ the vector of the upper bound of the coefficients of x_i in the objective function δ
$U = (\mathbf{c}_u^-\mathbf{x}, \mathbf{c}_u\mathbf{x}, \mathbf{c}_u^+\mathbf{x})$ the row vector of objective functions u with, respectively, the lower bound, the central and the upper bound of the objective coefficients as its elements
$V = (\mathbf{c}_v^-\mathbf{x}, \mathbf{c}_v\mathbf{x}, \mathbf{c}_v^+\mathbf{x})$ the row vector of objective functions v with, respectively, the lower bound, the central and the upper bound of the objective coefficients as its elements
$W = (\mathbf{c}_w^-\mathbf{x}, \mathbf{c}_w\mathbf{x}, \mathbf{c}_w^+\mathbf{x})$ the row vector of objective functions w with, respectively, the lower bound, the central and the upper bound of the objective coefficients as its elements
$Y = \left(\mathbf{c}_y^-\mathbf{x}, \mathbf{c}_y\mathbf{x}, \mathbf{c}_y^+\mathbf{x}\right)$ the row vector of objective functions y with, respectively, the lower bound, the central and the upper bound of the objective coefficients as its elements
$Z = (\mathbf{c}_z^-\mathbf{x}, \mathbf{c}_z\mathbf{x}, \mathbf{c}_z^+\mathbf{x})$ the row vector of objective functions z with, respectively, the lower bound, the central and the upper bound of the objective coefficients as its elements
$\overline{c_{\delta,c_i,x_i}}$ the degree of membership function which gives the degree of membership to the value of the decision variable x_i in the objective function $\delta$to the set of value represented by the term "about c_i"
$\delta_1 = (\mathbf{c}_\delta - \mathbf{c}_\delta^-)\mathbf{x}$ the difference between the objective function $\delta$with central coefficients and its corresponding function with lower bound coefficients
$\delta_2 = \mathbf{c}_\delta\mathbf{x}$ the objective function δ with central coefficients
$\delta_3 = (\mathbf{c}_\delta^+ - \mathbf{c}_\delta)\mathbf{x}$ the difference between the objective function δ with central coefficients and its corresponding function with upper bound coefficients
$\delta_j^{\max}$ the maximum value of the function δ_j in its feasible region
$\delta_j^{\min}$ the minimum value of the function δ_j in its feasible region
μ_{δ_j} the degree of satisfaction achieved by the value of the function δ_j
α the minimum value of μ_{δ_j} in their joint feasible region

has received much attention as it is an important variable required in the determination of the cost effectiveness of new technologies (Morgan & Daniels, 2001). For example, Kee and Schmidt (2000) develop a general model of the product-mix decision considering Theory of Constraints (TOC) and Activity-Based Costing (ABC). Petrovic-Lazarevic and Abraham (2003) make an attempt to point to a significance of applying a fuzzy approach to multi-objective decision methods in the process of organising business activities. The analysis and modelling of the construction industry problem presented in Petrovic-Lazarevic and Abraham (2003) is based on both linear objective functions and constraints in a form of linear membership functions.

Product-mix is also solved with a process known as Theory of Constraints (Goldratt, 1990; Goldratt, 1993). TOC gives a model for determining product-mix under constraint resources. A vast pool of research has been conducted to date on product-mix problem and its solution through TOC (Luebbe & Finch, 1992; Lee & Plenert, 1993; Fredendall & Lea, 1997; Plenert, 1993; Balakrishnan & Cheng, 2000; Hsu & Chung, 1998). For example, Bhattacharya, Vasant, Sarkar, and Mukherjee (2008) have utilised a fully fuzzified intelligent LP-model to determine the product-mix solution under TOC. Bhattacharya and Vasant (2007), Bhattacharya and Vasant (2006) outline a procedure for soft-sensing of level-of-satisfaction in TOC product-mix decision heuristic using robust fuzzy-LP. Other product-mix decision models are reported by Vasant and Bhattacharya (2005), Vasant and Bhattacharya (2005). It will be worthwhile to mention that Susanto, Vasant, and Bhattacharya (2006b) report a "Compromise Linear Programming having Fuzzy Objective Function Coefficients" (CLPFOFC) with fuzzy sensitivity. The work illustrates a chocolate manufacturing firm's product-mix decision using their CLPFOFC model. Susanto, Bhattacharya, Vasant, and and Suryadi (2006a), Susanto, Vasant, and Bhattacharya (2006c), Susanto, Bhattacharya, and Vasant (2006d) deal with product-mix decision models with both triangular and non-linear membership functions.

Several researchers studied the multi-objective formulation of decision models. According to Dyson (1980), fuzzy programming models should not be treated as a new contribution to multiple-objective decision-making methods, but rather as a lead to new conventional decision methods. "Support for this thesis would require examples of new and affective fuzzy inspired multi-criteria methods" (Dyson, 1980). But the present CFMOLP model delineated in the next sections, because of its synergistic effects, will reveal that the notion of Dyson (1980) is outdated. Many prominent researchers like Zimmermann (1985) have studied fuzzy LP model formulation and its solution.

Decision-making through fuzzy LP approach has been addressed in Zimmermann (1985), Dubois and Prade (1980), Yager, Ovchinikov, Tong, and Nguyen (1987), Ross (1995), Klir and Yuan (1995). Buckley, Feuring, and Hayashi (2001) report a multi-objective "fully fuzzified LP" methodology. They transform a MOLP problem into a single objective fuzzy LP problem and an evolutionary algorithm is reported in order to generate un-dominated set. Triangular fuzzy numbers are used in their solution. Another fuzzy approach – a generalization of max-min, averaging and two-phase method – is delineated by Chen and Chou (1996) to solve MOLP