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Interactive fuzzy random two-level linear programming through fractile criterion optimization

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In this paper, assuming cooperative behavior of the decision makers, solution methods for decision making problems in hierarchical organizations under fuzzy random environments are considered. To deal with the formulated two-level linear programming problems involving fuzzy random variables, α-level sets of fuzzy random variables are introduced and an α-stochastic two-level linear programming problem is defined for guaranteeing the degree of realization of the problem. Taking into account the vagueness of judgments of decision makers, fuzzy goals are introduced and the α-stochastic two-level linear programming problem is transformed into the problem to maximize the satisfaction degree for each fuzzy goal. Through the use of the fractile criterion optimization model, the transformed stochastic two-level programming problem can be reduced to a deterministic one. Interactive fuzzy programming to obtain a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented. It is shown that all of the problems to be solved in the proposed interactive fuzzy programming can be easily solved by the simplex method, the sequential quadratic programming or the combined use of the bisection method and the sequential quadratic programming. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method.

1. Introduction

Fuzzy random variables, first introduced by Kwakernaak [1], have been developing in various ways [2–4]. An overview of the developments of fuzzy random variables was found in the article of Gil et al. [5]. Studies on linear programming problems with fuzzy random variable coefficients, called fuzzy random linear programming problems, were initiated by Wang and Qiao [6], Qaio et al. [7] as seeking the probability distribution of the optimal solution and optimal value. Optimization models for fuzzy random linear programming problems were first considered by Luhandjula et al. [8,9] and further developed by Liu [10,11] and Rommelfanger [12]. A brief survey of major fuzzy stochastic programming models was found in the paper by Luhandjula [13]. As we look at recent developments in the fields of fuzzy random programming, we can see continuing advances [14–19,12,20–22].

However, decision making problems in hierarchical managerial or public organizations are often formulated as two-level mathematical programming problems [23]. In the context of two-level programming, the decision maker at the upper level first specifies a strategy, and then the decision maker at the lower level specifies a strategy so as to optimize the objective with full knowledge of the action of the decision maker at the upper level. In conventional multi-level mathematical programming models employing the solution concept of Stackelberg equilibrium, it is assumed that there

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is no communication among decision makers, or they do not make any binding agreement even if there exists such communication [24–31]. Compared with this, for decision making problems, for example, in decentralized large firms with divisional independence, it is quite natural to suppose that there exist communication and some cooperative relationship among the decision makers [23].

Lai [32] and Shih et al. [33] proposed solution concepts for two-level linear programming problems or multi-level ones such that decisions of decision makers in all levels are sequential and all of the decision makers essentially cooperate with each other. In their methods, the decision makers identify membership functions of the fuzzy goals for their objective functions, and in particular, the decision maker at the upper level also specifies those of the fuzzy goals for the decision variables. The decision maker at the lower level solves a fuzzy programming problem with a constraint with respect to a satisfactory degree of the decision maker at the upper level. Unfortunately, there is a possibility that their method leads a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and those of the decision variables. In order to overcome the problem in their methods, by eliminating the fuzzy goals for the decision variables, Sakawa et al. have proposed interactive fuzzy programming for two-level or multi-level linear programming problems to obtain a satisfactory solution for decision makers [34,35]. Subsequent works on two-level or multi-level programming have appeared [36–44,23].

Under these circumstances, in this paper, assuming cooperative behavior of the decision makers, we consider solution methods for decision making problems in hierarchical organizations under fuzzy random environments. To deal with the formulated two-level linear programming problems involving fuzzy random variables, α-level sets of fuzzy random variables are introduced and an α-stochastic two-level linear programming problem is defined for guaranteeing the degree of realization of the problem. Taking into account the vagueness of judgments of decision makers, fuzzy goals are introduced [45] and the α-stochastic two-level linear programming problem is transformed into the problem to maximize the satisfaction degree for each fuzzy goal. Following the fractile criterion optimization model [46], the transformed stochastic two-level programming problem can be reduced to a deterministic one. Interactive fuzzy programming to obtain a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented. It is shown that all of the problems to be solved in the proposed interactive fuzzy programming can be easily solved by the simplex method, the sequential quadratic programming or the combined use of the bisection method and the sequential quadratic programming. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method.

2. Fuzzy random two-level linear programming problems

Fuzzy random variables, first introduced by Kwakernaak [1], have been defined in various ways [1,3,2,4]. For example, as a special case of fuzzy random variables given by Kwakernaak, Kruse and Meyer [2] defined a fuzzy random variable as follows.

Definition 1 (*Fuzzy Random Variable*)**.** Let (Ω, B, P) be a probability space, $F(\mathcal{R})$ the set of fuzzy numbers with compact supports and X a measurable mapping $\Omega \to F(\mathcal{R})$. Then X is a fuzzy random variable if and only if given $\omega \in \Omega$, $X_\alpha(\omega)$ is a random interval for any $\alpha \in (0, 1]$, where $X_\alpha(\omega)$ is an α-level set of the fuzzy set $X(\omega)$.

Although there exist some minor differences in several definitions of fuzzy random variables, fuzzy random variables are considered to be random variables whose observed values are fuzzy sets.

In this paper, we deal with two-level linear programming problems involving fuzzy random variable coefficients in objective functions formulated as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{11}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{12}\boldsymbol{x}_2 \\ \underset{\text{for DM2}}{\text{minimize}} & z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{21}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{22}\boldsymbol{x}_2 \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \boldsymbol{b} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\}. \tag{1}$$

It should be emphasized here that randomness and fuzziness of the coefficients are denoted by the "dash above" and "wave above" i.e., "$\bar{}$" and "$\tilde{}$", respectively. In (1), $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the decision maker at the upper level (DM1), $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the decision maker at the lower level (DM2), $z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the objective function for DM1 and $z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the objective function for DM2. Elements $\tilde{\bar{C}}_{ljk}$, $k = 1, 2, \ldots, n_j$ of coefficient vectors $\tilde{\bar{\boldsymbol{C}}}_{lj}$, $l = 1, 2, j = 1, 2$ are fuzzy random variables, and their realized values $\tilde{\bar{C}}_{ljk}(\omega)$ for each elementary event $\omega \in \Omega$ are characterized by the membership function

$$\mu_{\tilde{\bar{C}}_{ljk}(\omega)}(\tau) = \begin{cases} L\left(\dfrac{\bar{d}_{ljk}(\omega) - \tau}{\beta_{ljk}}\right), & \text{if } \tau \le \bar{d}_{ljk}(\omega) \\ R\left(\dfrac{\tau - \bar{d}_{ljk}(\omega)}{\gamma_{ljk}}\right), & \text{otherwise,} \end{cases}$$