

Fuzzy linear programming under interval uncertainty based on IFS representation

Dipti Dubey*, Suresh Chandra, Aparna Mehra

Department of Mathematics, Indian Institute of Technology, Hauz Khas, New Delhi 110016, India

Received 11 June 2010; received in revised form 19 September 2011; accepted 20 September 2011

Available online 24 September 2011

Abstract

The equivalence between the interval-valued fuzzy set (IVFS) and the intuitionistic fuzzy set (IFS) is exploited to study linear programming problems involving interval uncertainty modeled using IFS. The non-membership of IFS is constructed with three different viewpoints viz., optimistic, pessimistic, and mixed. These constructions along with their indeterminacy factors result in S-shaped membership functions in the fuzzy counterparts of the intuitionistic fuzzy linear programming models. The solution methodology of Yang et al. [45], and its subsequent generalization by Lin and Chen [33] are used to compute the optimal solutions of the three fuzzy linear programming models.

© 2011 Elsevier B.V. All rights reserved.

Keywords: Fuzzy linear programming; Interval uncertainty; Intuitionistic fuzzy sets; Intuitionistic fuzzy goals; S-shaped membership function

1. Introduction

Fuzzy set (FS) theory [46] has been extensively used to capture linguistic uncertainty in decision making problems particularly in optimization problems. In fuzzy sets, the membership degree of an element in $[0, 1]$ expresses the degree of belongingness of an element to a fuzzy set. However, no objective procedure is available for the experts to assign the crisp membership degrees to the elements in a FS. In such a case, it is more reasonable to represent the membership degree of each element to the FS by means of an interval. It was suggested by Zadeh [47] to alleviate the fuzzy set theory to the *interval-valued fuzzy set* theory. An IVFS is a fuzzy set in which the membership degree is assumed to belong to an interval. On the other hand, in early 1980s, Atanassov [2] introduced another extension of Zadeh's FS namely the intuitionistic fuzzy set. IFS assigns to each element of the universe both a degree of membership and the degree of non-membership, which are more or less independent, related only by the constraint that the sum of two degrees must not exceed one. Subsequently, in [2,3,5,6] Atanassov developed the IFS theory. Although the two extensions, IVFS and IFS, were introduced independently yet surprisingly the two have been shown to be equivalent first in [7] and later in [12,15,18]. This framework forms the background of our present study wherein we shall be viewing the interval uncertainty (in the IVFS) using IFS representation.

* Corresponding author.

E-mail addresses: diptidubey@gmail.com (D. Dubey), chandras@maths.iitd.ac.in (S. Chandra), apmehra@maths.iitd.ac.in (A. Mehra).

Last two decades had seen the IFS theory crossing some milestones with its own share of controversy regarding its nomenclature [4,6,18,22]. The conflict has been mainly due to the already established field of intuitionistic logic (IL). Atanassov's IFS theory and the IL differ in their mathematical structure and treatment thereby making it confusing to use the same terminology for two different concepts. But seeing the statistical data of research publications and defended theses in the IFS theory, Atanassov [4] pointed out that the change of name would lead to a terminological chaos. However, Dubois et al. [18] disagreed with his arguments stating that it would be crucial and helpful for further development of Atanassov's IFS theory to refrain from this name. They also suggested to call intuitionistic fuzzy sets as Atanassov's intuitionistic fuzzy sets or just the bipolar fuzzy sets. A bipolar fuzzy set is a pair of fuzzy sets, one of which represent positive and the other represents the negative aspects of the given information, see [19,20]. Some other extensions of fuzzy set theory have also been reported in literature, for example, vague sets, gray sets, to name a few, but all these concepts turned out to be equivalent to Atanassov's intuitionistic fuzzy sets or bipolar sets. For detailed links between these objects and adequate theorems which present the isomorphisms between them, one may refer to [15,16]. However, till now there is no consensus among scientists on final name for IFS. The researchers continue to use the name IFS, and some of the recent publications [5,25,26,28,31,42–44] are strong evidence for it. In the same spirit, we shall henceforth be using the same name IFS to be understood in the sense of Atanassov's IFS.

Despite certain difficulties, the IF sets have already been used for (in alphabetic order of applications) clustering [43], medical diagnosis [13], multicriteria decision making [26–31,35,42], pattern recognition [25,41], to name a few. Besides these, there is a rich theory associated with IFS, for instance, refer to [3,10,12,14,17] and the references there in.

The first serious attempt to use IFS in optimization problems was made by Angelov [1] who proposed an optimization model by considering degrees of rejection of objective(s) and constraints together with their degrees of acceptance. He formulated an intuitionistic fuzzy optimization (IFO) model by adopting the approach of maximizing the degree of acceptance of intuitionistic fuzzy (IF) objective(s) and of constraints and minimizing the degree of rejection of IF objective(s) and constraints. Subsequently he formulated a crisp optimization problem using the IF aggregation operators in conjunction with the Bellman and Zadeh's [9] extension principle. Recently, in [44], Yager pointed out the difficulty in using a straight away extension of the Bellman and Zadeh's extension principle for aggregating IF decisions. He also suggested an alternative approach to choose the optimal decision while preserving the spirit of the Bellman and Zadeh's extension principle. We shall be describing these issues in brief in the section to follow. The corrective measure suggested by Yager [44] inspired us to modify the IFO model of [1] to propose new models for optimization problems in setup in intuitionistic fuzzy scenario.

The paper is planned as follows. In Section 2, we present a set of concepts from IFS theory which facilitate further discussion. In Section 3, a general framework for fuzzy linear programming problems under interval uncertainty is explained. We provide three different interpretations for IF inequality, namely IF essentially greater than equal to, by constructing the non-membership function in three different ways viz., optimistic, pessimistic, and mixed. In Section 4, we develop the corresponding three IFO models and formulated their fuzzy counterparts. We recognize that the membership functions of the resultant fuzzy optimization problems are S-shaped, thereby making us to apply the methods for solving fuzzy programming problems with piecewise linear S-shaped membership functions. In the present work we have used the methods proposed by Yang et al. [45] and Lin and Chen [33] to solve fuzzy programming problems with S-shaped membership functions by modeling their corresponding crisp optimization problems. Section 5 highlights some important issues needing attention, while Section 6 concludes the discussion.

2. Preliminaries

Let X denotes the universe set.

An IVFS \mathcal{A} is defined by a function $\mu_{\mathcal{A}}$ from X to the set of closed subintervals in $[0, 1]$, say, $\mu_{\mathcal{A}}(x) = [\mu_1(x), \mu_2(x)]$, $x \in X$.

An IFS \mathcal{A} assigns to each element x of the universe X a membership degree $\mu_{\mathcal{A}}(x) \in [0, 1]$ and a non-membership degree $\nu_{\mathcal{A}}(x) \in [0, 1]$ such that $\mu_{\mathcal{A}}(x) + \nu_{\mathcal{A}}(x) \leq 1$. Obviously, when $\mu_{\mathcal{A}}(x) + \nu_{\mathcal{A}}(x) = 1$, for all elements $x \in X$, the traditional fuzzy set concept is recovered. An IFS \mathcal{A} is mathematically represented as $\{(x, \mu_{\mathcal{A}}(x), \nu_{\mathcal{A}}(x)) | x \in X\}$.

Clearly, the aforementioned notions are mathematically isomorphic by taking $\mu_1(x) = \mu_{\mathcal{A}}(x)$, $\mu_2(x) = 1 - \nu_{\mathcal{A}}(x)$, $x \in X$.

The standard intersection of two IFS \mathcal{A} and \mathcal{B} is an IFS \mathcal{C} whose membership and non-membership functions are respectively defined as $\mu_{\mathcal{C}}(x) = \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{B}}(x)\}$ and $\nu_{\mathcal{C}}(x) = \max\{\nu_{\mathcal{A}}(x), \nu_{\mathcal{B}}(x)\}$, while the standard union of

متن کامل مقاله

دریافت فوری ←

ISIArticles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات