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Limit analysis of FRP strengthened masonry arches via nonlinear and linear programming

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ABSTRACT

The collapse load of masonry arches strengthened with FRP materials is determined. The arch is made of quadrangular blocks and the nonlinearity of the problem (no-tension material, frictional sliding and crushing) is concentrated at the interface between the blocks. Two methods are used to solve the problem. In the first method, a nonlinear programming problem (NLP) is formulated and is solved by using the successive quadratic programming algorithm (SQP) and combinatorial analysis. This method finds the optimal solution in the analysed cases. In the second method, a linear programming problem (LP) is formulated and is solved with classical techniques. LP approximates the optimal solution to any desired degree of accuracy. Although the number of variables of LP is much larger than that of NLP, LP process time can result much lower than NLP process time. Numerical examples are provided in order to show the advantages of the two methods and the effectiveness of FRP strengthening for different arch geometries.

1. Introduction

In civil engineering, FRP materials are usually used to strengthen masonry and concrete constructions. The design of the FRP strengthening of masonry structures is also supported by numerical and symbolic analyses. In fact, the load carrying capacity of masonry structures without or with FRP reinforcement can be determined with limit analysis. In [1], the masonry body is discretized with finite elements and the kinematic theorem of limit analysis is applied by assuming a kinematically admissible velocity field based on the finite element discretization. In many cases, masonry constructions are made of bricks and mortar joints: these structures can be modelled as the union of a finite number of blocks and the nonlinearity of the problem (no-tension material, frictional sliding, etc.) is concentrated at the interface between the blocks. The load carrying capacity of this block model is also determined with limit analysis [2–6]. Specifically, unilateral contact and nonassociative flow rules for frictional sliding between blocks have been considered in [3–5]. The formulation [6] takes into account a finite compressive strength of masonry as well as nonassociative frictional sliding; moreover, a tie element is added to model typical strengthening techniques such as metal bars and FRP strips. In [3,4,6], a nonlinear programming problem is solved for determining the collapse load of discrete block systems, while an iterative procedure which involves the successive solution of linear programming sub-problems is presented in [5]. Recently, second-order cone programming and semidefinite programming have been efficiently used for the limit analysis of cohesive–frictional materials [7,8]. The block method is not computationally expensive and is adapted in this work for determining the collapse load of masonry arches with externally bonded reinforcement (EBR). Associative flow rule is adopted for frictional sliding, as it has been observed that, in many single ring arch problems, the collapse load provided by associative friction is the same provided by non-associative friction [5]. The compressive strength of masonry is considered finite and the consequent crushing in the collapse mechanism is modelled by introducing a modified hinge mechanism [9,10], where hinges can form at internal or boundary points of an interface between two adjacent blocks that penetrate into each other. A perfect bond between EBR and masonry is assumed (see also [11–13]) and therefore the proposed model can be adopted in cases where debonding does not occur. Debonding can be prevented by applying anchor spikes [12] and by increasing the mechanical properties and the area of EBR–masonry interface; moreover, FRP debonding is more likely to occur at the intrados of an arch than at the extrados. In the kinematic approach used to solve the problem, EBR is modelled by imposing suitable restrictions to the kinematically admissible velocity field [14].

Solving a nonlinear programming problem (NLP) is a difficult task [3,4] and many authors [5,15,16] propose alternative limit analysis procedures based on the solution of a linear

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programming problem (LP). In [15,16], LP is used for the limit analysis of very complex problems characterized by unreinforced and FRP-reinforced masonry structures with single and double curvature shells and different failure criterions adopted for bricks and mortar. In the present work, we use two methods in order to solve the limit analysis problem. In the first method, the collapse load is determined by solving a NLP, where the objective function to minimize is nonlinear and the variables are subject to linear and nonlinear constraints. The NLP is solved by using the successive quadratic programming algorithm (SQP) [17]. The main difficulty of this iterative method is to find a good starting point, i.e. the values assigned to the variables of the NLP in the first iteration of the method. In fact, the SQP solution may depend on the choice of the starting point. In this work, a combinatorial analysis is used in order to find the starting point such that an optimal solution is provided by SQP. The CPU time for solving a combinatorial analysis increases very much with increasing the number of blocks. For this reason, a second method is proposed to determine the collapse load. Within this method, a LP is formulated and is solved with classical techniques. Although the number of variables of LP is much larger than that of NLP, LP process time can result much lower than NLP process time. Moreover, LP finds the optimal solution to any desired degree of accuracy. The LP solution is used to verify the accuracy of the NLP solution. Numerical examples are provided in order to show the advantages of the two methods and the effectiveness of FRP strengthening for different arch geometries.

2. Nonlinear programming problem (NLP)

The limit analysis problem is formulated for unreinforced arches in Section 2.1 and for arches with EBR in Section 2.2. These are nonlinear programming problems which are solved by using the successive quadratic programming algorithm explained in Section 2.3.

2.1. Unreinforced arch

The structure under consideration is a plane arch defined by a finite number of quadrangular elements (blocks). Prior to collapse, the structure occupies a constant reference configuration in the ξ,η -plane (Fig. 1). In the reference configuration, a side of the block i-1 occupies the same position of a side of the block i: the first side is denoted by $i0$ and the second side by $i1$. In the reference configuration, an interface i is the common part of two adjacent blocks and it coincides with sides $i0$ and $i1$. The number of interfaces is n_I and the length of an interface i is denoted by t_i. The position vector of the middle point of an interface i is denoted by $\mathbf{r}_i$. The middle point of a side $i\alpha$ (α = 0, 1) is denoted by $C_{i\alpha}$.

A block i can translate or rotate with respect to a block i-1, as illustrated in Figs. 2 and 3. The translation or rotation of a block i with respect to a block i-1 is called relative. In Fig. 2, the relative translation parallel to an interface i has velocity $\pm s_i^\pm$ (sliding), where $s_i^\pm \geqslant 0$, and the relative translation perpendicular to an interface i has velocity $\mu_i s_i^\pm$ (dilatancy). Dilatancy always accompanies sliding if μ_i is greater than zero. μ_i also represents the coefficient of friction at an interface i.

The centre of relative rotation is called hinge and is a point of the interface (Fig. 3). The hinge depth is the distance between the hinge and the end-point of the interface where block i-1 and block i interpenetrate. For an anticlockwise relative rotation, the hinge depth and the position vector of the hinge at a generic interface i are denoted by t_i^+ and $\mathbf{r}_i^+$, respectively. For a clockwise relative rotation, the hinge depth and the position vector of the hinge at a generic interface i are denoted by t_i^- and $\mathbf{r}_i^-$, respectively. The vector $\mathbf{r}_i^\pm$ is defined by

$$\mathbf{r}_i^\pm = \mathbf{r}_i \pm \mathbf{d}_i(t_i/2 - t_i^\pm), \tag{1}$$

where $\mathbf{d}_i$ is the unit vector parallel to an interface i and is oriented from the intrados to the extrados. For a generic interface i, the velocity of anticlockwise and clockwise relative rotations are denoted by φ_i^+ and $-\varphi_i^-$, respectively, where $\varphi_i^\pm \geqslant 0$. The kinematics of the model is defined by

$$\mathbf{v}_{i\alpha} = \sum_{j=1}^{i-1+\alpha} \Big(\mu_j\left(s_j^+ + s_j^-\right)\mathbf{d}_j^\perp + \left(s_j^+ - s_j^-\right)\mathbf{d}_j + \boldsymbol{\varphi}_j^+ \times \left(\mathbf{r}_i - \mathbf{r}_j^+\right) - \boldsymbol{\varphi}_j^- \times \left(\mathbf{r}_i - \mathbf{r}_j^-\right) + \boldsymbol{\varphi}_j^+ - \boldsymbol{\varphi}_j^-\Big), \tag{2}$$

where $\mathbf{d}_j=[d_{j\xi},d_{j\eta},0]^T$, $\mathbf{d}_j^\perp=[d_{j\eta},-d_{j\xi},0]^T$, $\boldsymbol{\varphi}_j^\pm=[0, 0, \varphi_j^\pm]^T$ and α = 0,1. The first and second components of $\mathbf{v}_{i\alpha}$ are the horizontal and vertical absolute velocities of a point $C_{i\alpha}$, respectively; the third component of $\mathbf{v}_{i\alpha}$ is the velocity of absolute rotation of a side $i\alpha$.

Dead and live loads are applied at point $C_{i\alpha}$ (α = 0, 1) and are defined by $\mathbf{p}_{i\alpha}$ and $\mathbf{q}_{i\alpha}$, respectively. The first and second components of $\mathbf{p}_{i\alpha}$ (or $\mathbf{q}_{i\alpha}$) are the horizontal and vertical concentrated forces, respectively; the third component of $\mathbf{p}_{i\alpha}$ (or $\mathbf{q}_{i\alpha}$) is the couple applied at a side $i\alpha$. It is assumed that the arch is subject to dead load $\mathbf{p}_{i\alpha}$ and variable load $\lambda\mathbf{q}_{i\alpha}$, where λ is a multiplier of live load $\mathbf{q}_{i\alpha}$. The problem is to find the collapse multiplier λ_c, which is the maximum value of λ satisfying the governing equations of the problem. From the virtual power principle, the collapse multiplier λ_c of live load is the minimum of kinematically admissible multipliers λ and it is given by

$$\lambda_c = \min_V(P_\sigma - P_G), \tag{3}$$

where P_G is the virtual power of dead load, P_σ is the internal virtual power and V is the set of kinematically admissible velocity fields such that the virtual power P_Q of live load is equal to one. The virtual powers of dead and live loads are defined by

$$P_G = \sum_{i=1}^{n_I} \sum_{\alpha=0,1} \mathbf{v}_{i\alpha}\mathbf{p}_{i\alpha}, \quad P_Q = \sum_{i=1}^{n_I} \sum_{\alpha=0,1} \mathbf{v}_{i\alpha}\mathbf{q}_{i\alpha}, \tag{4}$$

respectively. The internal virtual power is defined by

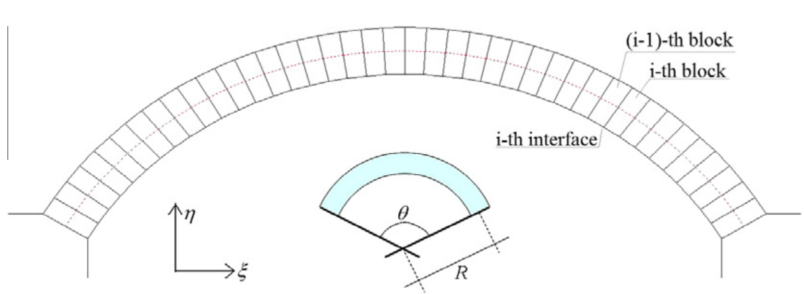

Fig. 1. An arch in the reference configuration.