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Alternative mixed integer linear programming models for identifying the most efficient decision making unit in data envelopment analysis ☆,☆☆

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ABSTRACT

A mixed integer linear model for selecting the best decision making unit (DMU) in data envelopment analysis (DEA) has recently been proposed by Foroughi [Foroughi, A. A. (2011a). A new mixed integer linear model for selecting the best decision making units in data envelopment analysis. *Computers and Industrial Engineering, 60*(4), 550–554], which involves many unnecessary constraints and requires specifying an assurance region (AR) for input weights and output weights, respectively. Its selection of the best DMU is easy to be affected by outliers and may sometimes be incorrect. To avoid these drawbacks, this paper proposes three alternative mixed integer linear programming (MILP) models for identifying the most efficient DMU under different returns to scales, which contain only essential constraints and decision variables and are much simpler and more succinct than Foroughi's. The proposed alternative MILP models can make full use of input and output information without the need of specifying any assurance regions for input and output weights to avoid zero weights, can make correct selections without being affected by outliers, and are of significant importance to the decision makers whose concerns are not DMU ranking, but the correct selection of the most efficient DMU. The potential applications of the proposed alternative MILP models and their effectiveness are illustrated with four numerical examples.

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1. Introduction

Data envelopment analysis (DEA), developed by Charnes, Cooper, and Rhodes (1978), is a useful yet practical methodology for efficiency assessment based on multiple inputs and multiple outputs. Through efficiency measurement, DEA can identify efficient and inefficient decision making units (DMUs). Efficient DMUs are those with efficiency of one, while inefficient DMUs are those with their efficiencies being less than one. Efficient DMUs can be further discriminated through a ranking approach like super-efficiency procedure (Andersen & Petersen, 1993) or cross-efficiency evaluation (Doyle & Green, 1994; Sexton, Silkman, & Hogan, 1986) to produce a full ranking or select the most efficient DMU.

Sometimes, DMU ranking is not a main concern. For example, in DEA applications such as robot selection (Baker & Talluri, 1997; Khouja, 1995), flexible manufacturing system (FMS) selection (Shang & Sueyoshi, 1995), and computer numerical control (CNC) machine selection (Sun, 2002), what the decision maker (DM) is concerned about is the selection of the most efficient DMU, rather than DMU ranking. So, in these situations, there is no need to measure the performance of every DMU and a very practical way is to develop a model to find the most efficient DMU directly without assessing the performances of the other DMUs.

There have been several attempts in DEA literature to develop DEA models for finding the most efficient DMU. For example, Amin and Toloo (2007) proposed an integrated DEA model, which is in nature a mixed integer linear programming, to find the most efficient DMU in two steps. The first step is to find a maximum epsilon value for input and output weight variables and then the second step is to solve the integrated DEA model with the predetermined maximum epsilon value. This integrated model was later applied to find the most efficient association rule in data mining by Toloo, Sohrabi, and Nalchigar (2009) and extended to the situation of variable returns to scale (VRS) by Toloo and Nalchigar (2009). Amin (2009) found the integrated model of Amin and Toloo (2007) flawed, which might produce more than one efficient DMU, and he thus further suggested a mixed integer nonlinear model. This mixed integer nonlinear model, however, was found infeasible in some cases by Foroughi (2011a).

To resolve the infeasibility problem of the mixed integer nonlinear model, Foroughi (2011a) proposed a mixed integer linear model to find the most efficient DMU from the perspective of super efficiency. This mixed integer linear model, however, is found involving

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too many unnecessary constraints and requiring the specification of assurance regions (ARs) for input and output weights to avoid zero weights. It is also found that this mixed integer linear model proposed by Foroughi (2011a) is easy to be affected by outliers, leading to its selection of the best DMU being incorrect.

To eliminate these drawbacks and provide more methodological and model options for the decision maker, we propose in this paper three alternative mixed integer linear programming (MILP) models for finding the most efficient DMU under different returns to scales. In comparison with the existing DEA models for finding the most efficient DMU, the proposed alternative MILP models are more succinct, more practical and more reliable and contain only essential constraints and decision variables. More importantly, they can make the best use of input and output information without the need of specifying any assurance regions for input and output weights to avoid zero weights, can make correct selections without being affected by outliers, and are of significant importance to the decision makers whose concerns are not DMU ranking, but the correct selection of the most efficient DMU. Moreover, the proposed alternative MILP models also enhance the theory of DEA.

The rest of the paper is organized as follows: Section 2 briefly reviews existing models for finding the most efficient DMU and points out their drawbacks. Section 3 proposes the alternative MILP models under different returns to scales. Section 4 examines four numerical examples to show the potential applications of the proposed alternative MILP models and their effectiveness in finding the most efficient DMU. The paper concludes in Section 5.

2. Existing models for finding the most efficient DMU

Suppose that there are n DMUs to be evaluated in terms of m inputs and s outputs. Denote by x_{ij} $(i = 1, \ldots, m)$ and y_{rj} $(r = 1, \ldots, s)$ the input and output values of DMU_j $(j = 1, \ldots, n)$. To find the most efficient DMU, Amin and Toloo (2007) proposed the following integrated DEA model:

$$\begin{aligned}
&\text{Minimize} \quad M\\
&\text{Subject to} \quad M - d_j \geqslant 0, \quad j = 1,\ldots,n,\\
&\sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad J = 1,\ldots,n,\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0, \quad j = 1,\ldots,n,\\
&\sum_{j=1}^{n} d_j = n-1,\\
&0 \leqslant \beta_j \leqslant 1, d_j \in \{0,1\}, \quad j = 1,\ldots,n,\\
&u_r \geqslant \varepsilon^*, \quad r = 1,\ldots,s,\\
&w_i \geqslant \varepsilon^*, \quad i = 1,\ldots,m,
\end{aligned} \tag{1}$$

where ε^* is the maximum non-Archimedean infinitesimal determined by

$$\begin{aligned}
\varepsilon^* = &\text{Maximize} \quad \varepsilon\\
&\text{Subject to} \quad \sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad j = 1,\ldots,n,\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} \leqslant 0, \quad j = 1,\ldots,n,\\
&u_r - \varepsilon \geqslant 0, \quad r = 1,\ldots,s,\\
&w_i - \varepsilon \geqslant 0, \quad i = 1,\ldots,m.
\end{aligned} \tag{2}$$

It is claimed that the most efficient DMU corresponds to the DMU_j with optimal value $d_j^* = 0$. It is easy to see that the optimal value M^* is identically equal to one (i.e. $M^* \equiv 1$) no matter what values the other decision variables take due to the fact that $(n - 1)$ decision variables d_j always take the value of one. So, model (1) is ill-defined and there is no meaning to solve it. From model (2), it is also known that there is at least one DMU whose constraint $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} \leqslant 0$ is binding, i.e. $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} = 0$; otherwise, we can always make it tight by increasing the values of u_r $(r = 1, \ldots, s)$. In the case where there are multiple DMUs with binding constraints $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} = 0$, binary variable d_j for any one of them can take a zero value. So, model (1) will randomly choose one of them as the most efficient DMU and the selection is neither unique nor optimal. This amounts to randomly picking up one from DEA efficient units as the most efficient DMU. The true most efficient DMU has still not been identified correctly.

Toloo and Nalchigar (2009) extended the integrated DEA model (1) to VRS situation and proposed the following DEA model for finding the most BCC-efficient DMU:

$$\begin{aligned}
&\text{Minimize} \quad M\\
&\text{Subject to} \quad M - d_j \geqslant 0, \quad j = 1,\ldots,n,\\
&\sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad j = 1,\ldots,n,\\
&\sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0, \quad j = 1,\ldots,n,\\
&\sum_{j=1}^{n} d_j = n-1,\\
&0 \leqslant \beta_j \leqslant 1, d_j \in \{0,1\}, \quad j = 1,\ldots,n,\\
&u_r \geqslant \varepsilon^*, \quad r = 1,\ldots,s,\\
&w_i \geqslant \varepsilon^*, \quad i = 1,\ldots,m,\\
&u_0 \text{ is free in sign},
\end{aligned} \tag{3}$$

where ε^* is the maximum non-Archimedean infinitesimal determined by

$$\begin{aligned}
\varepsilon^* = &\text{Maximize} \quad \varepsilon\\
&\text{Subject to} \quad \sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad j = 1,\ldots,n,\\
&\sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} \leqslant 0, \quad j = 1,\ldots,n,\\
&u_r - \varepsilon \geqslant 0, \quad r = 1,\ldots,s,\\
&w_i - \varepsilon \geqslant 0, \quad i = 1,\ldots,m,\\
&u_0 \text{ is free in sign}.
\end{aligned} \tag{4}$$

Model (3) suffers from exactly the same drawbacks as model (1). In particular, it can choose any one of BCC-efficient DMUs as the most BCC-efficient by chance, depending on the solution method or software used for solving the problem (Foroughi, 2011b).

To find a single most efficient DMU, Amin (2009) proposed an improved integrated DEA model, as shown below:

$$\begin{aligned}
&\text{Minimize} \quad M\\
&\text{Subject to} \quad M - d_j \geqslant 0, \quad j = 1,\ldots,n,\\
&\sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad j = 1,\ldots,n,\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} + d_j = 0, \quad j = 1,\ldots,n,\\
&\sum_{j=1}^{n} \theta_j = n-1,\\
&\theta_j - d_j \beta_j = 0, \quad j = 1,\ldots,n,\\
&d_j \geqslant 0, \beta_j \geqslant 1, \theta_j \in \{0,1\}, \quad j = 1,\ldots,n,\\
&u_r \geqslant \varepsilon^*, \quad r = 1,\ldots,s,\\
&w_i \geqslant \varepsilon^*, \quad i = 1,\ldots,m,
\end{aligned} \tag{5}$$

where ε^* is the maximum non-Archimedean infinitesimal determined by model (2). It is claimed that the nonlinear constraints $\theta_j - d_j\beta_j = 0$

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