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A note on weak sharp minima in multicriteria linear programming☆

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ABSTRACT

In this note, the set of weak Pareto solutions of a multicriteria linear programming problem (MCLP, for short) is proved to be a set of weak sharp minima for another residual function of MCLP, i.e., the minimum of the natural residual functions of finitely many scalarization problems of MCLP, which is less than the natural residual function of MCLP. This can be viewed as a slight improvement of a result due to Deng and Yang. Some examples are given to illustrate these results.

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1. Introduction and preliminaries

Let $\mathbb{R}^p$ be a p-dimensional Euclidean space. Denote by $\mathbb{R}^p_+ = \{x = (x_1, \ldots, x_p) | x_i \geq 0, i = 1, \ldots, p\}$ and by

$$\triangle_p = \left\{ \xi = (\xi_1, \ldots, \xi_p) \in \mathbb{R}^p_+ \,\middle|\, \sum_{i=1}^{p} \xi_i = 1 \right\}$$

the unit simplex, where p is a given positive integer. In this paper, we always consider the max-norm in Euclidean space $\mathbb{R}^p$, i.e.,

$$\|x - y\| = \max_{1 \leq i \leq p} |x_i - y_i|,$$

where $x = (x_1, \ldots, x_p)$ and $y = (y_1, \ldots, y_p) \in \mathbb{R}^p$. Let $A \subseteq \mathbb{R}^p$ and $x \in \mathbb{R}^p$. Denote by $d(x, A)$ the distance from x to A, i.e., $d(x, A) = \inf_{a \in A} \|x - a\|$.

Consider the multicriteria linear programming problem (MCLP, for short) as follows:

$$\min\ Cx \quad \text{subject to } x \in P,$$

where $Cx = (\langle c_1, x\rangle, \ldots, \langle c_n, x\rangle)$, $c_i = (c_i^1, \ldots, c_i^m) \in \mathbb{R}^m$ $(i = 1, \ldots, n)$ and $P \subseteq \mathbb{R}^m$ is a nonempty polyhedral convex set.

A point $x^* \in P$ is called a weak Pareto solution of MCLP if,

$$Cx - Cx^* \notin -\operatorname{int} \mathbb{R}^n_+, \quad \forall x \in P,$$

or equivalently,

$$Cx - Cx^* \in \mathbb{R}^n_+ \setminus (-\operatorname{int} \mathbb{R}^n_+), \quad \forall x \in P.$$

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Denote by E_w the set of weak Pareto solutions of MCLP. In this paper, we always suppose that E_w is nonempty.

It is clear that $d(Cx, CE_w) = 0$ and $x \in P$ if and only if $x \in E_w$ and so $d(Cx, CE_w)$ serves as a natural residual function for MCLP. We say that E_w is a set of weak sharp minima for the function $d(Cx, CE_w)$ if, there exists $\tau > 0$ such that

$$d(x, E_w) \le \tau d(Cx, CE_w), \quad \forall x \in P.$$

The concept of a sharp minimum for real-valued functions was introduced in [1]. Weak sharp minima for real-valued functions, as a generalization of sharp minima, were introduced and investigated by Ferris [2]. Weak sharp minima play important roles in mathematical programming. It is well known that weak sharp minima are closely related to error bounds in convex programming, the sensitivity analysis of optimization problems and the convergence analysis of some algorithms (see, for example, [3–7]). In [8], Deng and Yang studied the existence of weak sharp minima in MCLP and proved that weak sharp minimality holds for the natural residual function $d(Cx, CE_w)$.

As it is well known that among solution approaches for MCLP, scalarization is one of the most analyzed topics at least from the computational point of view. By the well known structure of MCLP [9], there are finitely many vectors in the unit simplex such that E_w is the union of the sets of solutions of scalarization problems of MCLP related to such vectors. To this end, in this note, we prove that E_w is a set of weak sharp minima for another residual function of MCLP, i.e., the minimum of the natural residual functions of such finitely many scalarization problems of MCLP, which is shown to be less than the natural residual function $d(Cx, CE_w)$. This can be viewed as a slight improvement of a result due to Deng and Yang [8]. We will give some examples to illustrate these results.

2. Weak sharp minima for MCLP

Let $\xi \in \triangle_n$. Consider the following scalarization problem of MCLP (for short, $(\mathrm{SP})_\xi$):

$$\min \langle \xi, Cx \rangle \quad \text{subject to } x \in P,$$

or equivalently,

$$\min \left\langle \sum_{i=1}^{n} \xi_i c_i, x \right\rangle \quad \text{subject to } x \in P.$$

Denote by S_ξ the set of solutions to problem $(\mathrm{SP})_\xi$.

By the well known structure of MCLP [9], there are finitely many vectors $\xi(1), \ldots, \xi(r)$ of $\triangle_n$ such that $E_w = \cup_{k=1}^{r} S_{\xi(k)}$, where $S_{\xi(k)}$ is the set of solutions to problem $(\mathrm{SP})_{\xi(k)}$. Without loss of generality, we suppose that $S_{\xi(k)}$ is nonempty for each $k = 1, \ldots, r$.

Since the scalarization approach is one of the most analyzed topics at least from the computational point of view, it is important and useful to consider the residual function associated with a scalarization of MCLP.

For $x \in \mathbb{R}^m$, we have

$$\begin{aligned} d(x, E_w) &= \inf_{y \in E_w} \|x - y\| \\ &= \inf_{y \in E_w} \max_{1 \le j \le m} |x_j - y_j|, \\ \max_{1 \le k \le r} d(\langle \xi(k), Cx \rangle, \langle \xi(k), CE_w \rangle) &= \max_{1 \le k \le r} \inf_{y \in E_w} |\langle \xi(k), C(x - y) \rangle| \\ &= \max_{1 \le k \le r} \inf_{y \in E_w} \left| \sum_{i=1}^{n} \xi(k)_i \langle c_i, x - y \rangle \right| \end{aligned}$$

and

$$\begin{aligned} d(Cx, CE_w) &= \inf_{y \in E_w} \|C(x - y)\| \\ &= \inf_{y \in E_w} \max_{1 \le i \le n} |\langle c_i, x - y \rangle|. \end{aligned}$$

For $x \in \mathbb{R}^m$ and $k = 1, \ldots, r$, let

$$\Phi(k, x) = d(\langle \xi(k), Cx \rangle, \langle \xi(k), CE_w \rangle)$$

and

$$\Psi(k, x) = d(\langle \xi(k), Cx \rangle, \langle \xi(k), CS_{\xi(k)} \rangle).$$

Clearly, $\Psi(k, x)$ is the natural residual function of problem $(\mathrm{SP})_{\xi(k)}$.

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