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Stackelberg solutions for random fuzzy two-level linear programming through possibility-based probability model

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ABSTRACT

This paper considers computational methods for obtaining Stackelberg solutions to random fuzzy two-level linear programming problems. Assuming that the decision makers concerns about the probabilities that their own objective function values are smaller than or equal to certain target values, fuzzy goals of the decision makers for the probabilities are introduced. Using the possibility-based probability model to maximize the degrees of possibility with respect to the attained probability, the original random fuzzy two-level programming problems are reduced to deterministic ones. Extended concepts of Stackelberg solutions are introduced and computational methods are also presented. A numerical example is provided to illustrate the proposed method.

1. Introduction

In the real world, we often encounter situations where there are two or more decision makers in an organization with a hierarchical structure, and they make decisions in turn or at the same time so as to optimize their objective functions. Decision making problems in decentralized organizations are often modeled as Stackelberg games (Simaan and Cruz, 1973), and they are formulated as two-level mathematical programming problems (Shimizu et al., 1997; Sakawa and Nishizaki, 2009). In the context of two-level programming, the decision maker at the upper level first specifies a strategy, and then the decision maker at the lower level specifies a strategy so as to optimize the objective with full knowledge of the action of the decision maker at the upper level. In conventional multi-level mathematical programming models employing the solution concept of Stackelberg equilibrium, it is assumed that there is no communication among decision makers, or they do not make any binding agreement even if there exists such communication.

Computational methods for obtaining Stackelberg solutions to two-level linear programming problems are classified roughly into three categories: the vertex enumeration approach (Bialas and Karwan, 1984), the Kuhn–Tucker approach (Bard and Falk, 1982; Bard and Moore, 1990; Bialas and Karwan, 1984; Hansen et al., 1992), and the penalty function approach (White and Anandalingam, 1993). The subsequent works on two-level programming problems under noncooperative behavior of the decision makers have been appearing (Colson et al., 2005; Faisca et al., 2007; Gümüs and Floudas, 2001; Nishizaki and Sakawa, 2000; Nishizaki et al., 2003) including some applications to aluminium production process (Nicholls, 1996), pollution control policy determination (Amouzegar and Moshirvaziri, 1999), tax credits determination for biofuel producers (Dempe and Bard, 2001), pricing in competitive electricity markets (Fampa et al., 2008), supply chain planning (Roghanian et al., 2007) and so forth.

In order to deal with multiobjective problems (Sakawa, 1993, 2001) in hierarchical decision making, two-level multiobjective linear programming problems were formulated and a computational method for obtaining the corresponding Stackelberg solution was also developed (Nishizaki and Sakawa, 1999). Considering stochastic events related to hierarchical decision making situations, on the basis of stochastic programming models, two-level programming problems with random variables were formulated and algorithms for deriving the Stackelberg solutions were developed (Nishizaki et al., 2003). Furthermore, considering not only the randomness of parameters involved in objective functions and/or constraints but also the experts' ambiguous understanding of realized values of the random parameters, fuzzy random two-level linear programming problems were formulated, and computational methods for obtaining the corresponding Stackelberg solutions were also developed (Sakawa and Katagiri, 2012; Sakawa and Kato, 2009; Sakawa et al., in press; Sakawa et al., 2011).

From a viewpoint of ambiguity and randomness different from fuzzy random variables (Kwakernaak, 1978; Puri and Ralescu, 1986; Wang and Qiao, 1993), by considering the experts' ambiguous understanding of means and variances of random variables, a concept of random fuzzy variables was proposed, and

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mathematical programming problems with random fuzzy variables were formulated together with the development of a simulation-based approximate solution method (Liu, 2002).

Under these circumstances, in this paper, assuming noncooperative behavior of the decision makers, we formulate random fuzzy two-level linear programming problems. To deal with the formulated two-level linear programming problems involving random fuzzy variables, we assume that the decision makers concerns about the probabilities that their own objective function values are smaller than or equal to certain target values. By considering the imprecise nature of the human judgments, we introduce the fuzzy goals of the decision makers for the probabilities. Then, assuming that the decision makers are willing to maximize the degrees of possibility with respect to the attained probability, we consider the possibility-based probability model for random fuzzy two-level programming problems. Extended concepts of Stackelberg solutions are introduced. Computational methods for obtaining approximate Stackelberg solutions through particle swarm optimization are also presented. An illustrative numerical example demonstrates the feasibility and efficiency of the proposed method.

2. Random fuzzy variables

In the framework of stochastic programming, it is implicitly assumed that the uncertain parameter which well represents the stochastic factor of real systems can be definitely expressed as a single random variable. However, from the expert's experimental point of view, the experts may think of a collection of random variables to be appropriate to express stochastic factors rather than only a single random variables. In this case, reflecting the expert's conviction degree that each of random variables properly represents the stochastic factor, it would be quite reasonable to assign the different degrees of possibility to each of random variables. For handling such an uncertain parameter, a random fuzzy variable was defined by Liu (2002) as a function from a possibility space to a collection of random variables, which is considered to be an extended concept of fuzzy variable (Nahmias, 1978). It should be noted here that the fuzzy variables can be viewed as another way of dealing with the imprecision which was originally represented by fuzzy sets. Although we can employ Liu's definition, for consistently discussing various concepts in relation to the fuzzy sets, we define the random fuzzy variables by extending not the fuzzy variables but the fuzzy sets.

Definition 1 (*Random fuzzy variable*). Let Γ be a collection of random variables. Then, a random fuzzy variable $\tilde{\bar{C}}$ is defined by its membership function

$$\mu_{\tilde{\bar{C}}} : \Gamma \to [0, 1]. \tag{1}$$

In Definition 1, the membership function $\mu_{\tilde{\bar{C}}}$ assigns each random variable $\bar{\gamma} \in \Gamma$ to a real number $\mu_{\tilde{\bar{C}}}(\bar{\gamma})$. It should be noted here that if Γ is defined as $\mathbb{R}$, then (1) becomes equivalent to the membership function of an ordinary fuzzy set. In this sense, a random fuzzy variable can be regarded as an extended concept of fuzzy sets. On the other hand, if Γ is defined as a singleton $\Gamma = \{\bar{\gamma}\}$ and $\mu_{\tilde{\bar{C}}}(\bar{\gamma}) = 1$, then the corresponding random fuzzy variable $\tilde{\bar{C}}$ can be viewed as an ordinary random variable.

When taking account of the imprecise nature of the realized values of random variables, it would be appropriate to employ the concept of fuzzy random variables. However, it should be emphasized here that if mean and/or variance of random variables are specified by the expert as a set of real values or fuzzy sets, such uncertain parameters can be represented by not fuzzy random variables but random fuzzy variables.

As a simple example of random fuzzy variables, we consider a Gaussian random variable whose mean value is not definitely specified as a constant. For example, when some random parameter $\bar{\gamma}$ is represented by the Gaussian random variable $N(s_i, 10^2)$ where the expert identifies a set $\{s_1, s_2, s_3\}$ of possible mean values as $(s_1, s_2, s_3) = (90, 100, 110)$, if the membership function $\mu_{\tilde{\bar{C}}}$ is defined by

$$\mu_{\tilde{\bar{C}}}(\bar{\gamma}) = \begin{cases} 0.5 & \text{if } \bar{\gamma} \sim N(90, 10^2), \\ 0.7 & \text{if } \bar{\gamma} \sim N(100, 10^2), \\ 0.3 & \text{if } \bar{\gamma} \sim N(110, 10^2), \\ 0 & \text{otherwise}, \end{cases}$$

then $\tilde{\bar{C}}$ is a random fuzzy variable. More generally, when the mean values are expressed as fuzzy sets or fuzzy numbers, the corresponding random variable with the fuzzy mean is represented by a random fuzzy variable.

3. Problem formulation and transformed problems

Random fuzzy two-level linear programming problems are generally formulated as

$$\left.\begin{array}{l} \underset{\boldsymbol{x}_1}{\text{minimize}}\, z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{11}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{12}\boldsymbol{x}_2 \\ \text{where } \boldsymbol{x}_2 \text{ solves} \\ \underset{\boldsymbol{x}_2}{\text{minimize}}\, z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tilde{\bar{\boldsymbol{C}}}_{21}\boldsymbol{x}_1 + \tilde{\bar{\boldsymbol{C}}}_{22}\boldsymbol{x}_2 \\ \text{subject to} \quad A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leqslant \boldsymbol{b} \\ \qquad\qquad \boldsymbol{x}_1 \geqslant \boldsymbol{0},\ \boldsymbol{x}_2 \geqslant \boldsymbol{0}, \end{array}\right\} \tag{2}$$

where $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the decision maker at the upper level (DM1), $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the decision maker at the lower level (DM2), A_j, $j = 1, 2$ are $m \times n_j$ coefficient matrices, and $\boldsymbol{b}$ is an m dimensional column vector, and $z_l(\boldsymbol{x}_1, \boldsymbol{x}_2)$, $l = 1, 2$ are the objective functions for DMl, $l = 1, 2$, respectively.

Observing that the real data with uncertainty are often distributed normally, from the practical point of view, we assume that each of $\tilde{\bar{C}}_{ljk}$, $k = 1, 2, \ldots, n_j$ of $\tilde{\bar{\boldsymbol{C}}}_{lj}$, $l = 1, 2$, $j = 1, 2$ is the Gaussian random variable with fuzzy mean value $\tilde{M}_{ljk}$ which is represented by an L–R fuzzy number characterized by the membership function

$$\mu_{\tilde{M}_{ljk}}(\tau) = \begin{cases} L\left(\frac{m_{ljk} - \tau}{\alpha_{ljk}}\right) & \text{if } m_{ljk} \geqslant \tau \\ R\left(\frac{\tau - m_{ljk}}{\beta_{ljk}}\right) & \text{if } m_{ljk} < \tau, \end{cases} \tag{3}$$

where the shape functions L and R are nonincreasing continuous functions from $[0, \infty)$ to $[0, 1]$, m_{ljk} is the mean value, and α_{ljk} and β_{ljk} are positive numbers which represent left and right spreads. Fig. 1 illustrates an example of the membership function $\mu_{\tilde{M}_{ljk}}(\tau)$.

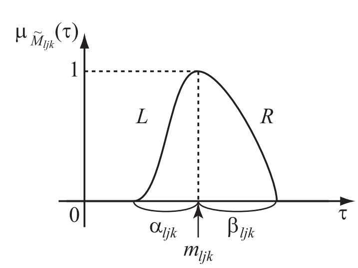

Fig. 1. An example of the membership function $\mu_{\tilde{M}_{ljk}}(\tau)$.