Applied Mathematical Modelling 36 (2012) 6301–6311

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Adjacency based method for generating maximal efficient faces in multiobjective linear programming

S. Krichen [a,b,*], H. Masri [b], A. Guitouni [c,d]

[a] Faculty of Law, Economics and Management, University of Jendouba, Avenue de l'UMA, 8189 Jendouba, Tunisia
[b] LARODEC Laboratory, Institut Supérieur de Gestion de Tunis, University of Tunis, Rue de la liberté, 2000 Bardo, Tunisia
[c] Command and Control Decision Support System Section, Defence Research and Development Canada (DRDC) Valcartier, 2459 Pie-XI North, Quebec (Quebec), Canada G3J 1X5
[d] Faculty of Business, University of Victoria, Victoria, British-Columbia, Canada

ARTICLE INFO

Article history:
Received 15 July 2010
Received in revised form 6 January 2012
Accepted 17 January 2012
Available online 9 February 2012

Keywords:
Multiobjective linear programming
Adjacency
Efficient set
Efficient faces
Outcome set

ABSTRACT

Multiobjective linear optimization problems (MOLPs) arise when several linear objective functions have to be optimized over a convex polyhedron. In this paper, we propose a new method for generating the entire efficient set for MOLPs in the outcome space. This method is based on the concept of adjacencies between efficient extreme points. It uses a local exploration approach to generate simultaneously efficient extreme points and maximal efficient faces. We therefore define an efficient face as the combination of adjacent efficient extreme points that define its border. We propose to use an iterative simplex pivoting algorithm to find adjacent efficient extreme points. Concurrently, maximal efficient faces are generated by testing relative interior points. The proposed method is constructive such that each extreme point, while searching for incident faces, can transmit some local informations to its adjacent efficient extreme points in order to complete the faces' construction. The performance of our method is reported and the computational results based on randomly generated MOLPs are discussed.

1. Introduction

Many real optimization problems can be modeled as a multiobjective linear programming (MOLP) problem, such as transportation [1] and inventory planning [2]. MOLPs are characterized by the simultaneous maximization of a set of objectives under a system of constraints, where both objective functions and constraints are linear, within continuous decision variables. Solving MOLPs yields to a set of efficient solutions belonging to a connected set of efficient faces. The set of all efficient solutions is the union of all maximal efficient faces. Several approaches for solving such problems have been proposed in the literature [3–5]. These methods employ different schemes for exploring the efficient set, and different ways of characterizing and locating the maximal efficient faces.

Recently, much more attention has been given to solving MOLPs in objective space. The earlier works of Dauer and Gallagher [6] and Benson [7] provided a new view for studying efficient solutions set in outcome space. Comparing to the decision based approaches, the outcome based search methods seem to give promissive computational results [8]. Yan et al. [9] presented a method for generating maximal efficient faces in combined decision-outcome space using the weight decomposition and with an easy and clear solution structure, they defined the set of efficient faces as a finite combination of

* Corresponding author at: Faculty of Law, Economics and Management, University of Jendouba, Avenue de l'UMA, 8189 Jendouba, Tunisia
E-mail addresses: saoussen.krichen@isg.rnu.tn (S. Krichen), masri_hela@yahoo.fr (H. Masri), Adel.Guitouni@drdc-rddc.gc.ca (A. Guitouni).

0307-904X/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
doi:10.1016/j.apm.2012.01.034

extreme points and rays. This representation was called the sum form. More details might be found is Section 3. Most of exiting outcome based methods, operate in two steps: generating efficient extreme points then locating efficient faces.

In this paper, we propose a new method for solving MOLPs in objective space, by combining the sum form approach with the adjacency information of efficient extreme points. The proposed method performs the neighborhood approach defined by Benson and Sun [10] in order to generate efficient extreme points. Simultaneously, the efficiency of certain combinations of these extreme points are tested by investigating relative interior points in order to find maximal efficient faces. The main idea is to design a local and constructive method. We associate to each extreme point x a list $L(x)$ to keep in memory the faces containing this point. At each iteration, we investigate a new extreme point. We start by exploring its adjacent efficient vertices. If a new efficient point x' is generated, then we test the efficiency of the convex hull defined by combining this new point x' with preexisting faces in $L(x)$. To do so, a relative interior point is tested. If a new face is encountered, we update the list $L(x')$ by adding this face. This procedure iterates until exploring all efficient extreme points.

The paper is structured as follows. In the next section, we present the multiobjective linear programming problem and some theoretical background. In Section 3, we review related literature. In Section 4, we present the proposed method for generating maximal efficient faces in outcome space. We provide empirical results in Section 5. A comparison with an existing method [5], as well as a computational analysis for medium and large sized problems are also discussed in the same section. The conclusion is presented in Section 6.

2. Problem description and fundamental results

A MOLP problem can be defined as:

$$\begin{aligned} \text{Min} \quad & Cx \\ \text{s.t} \quad & Ax \leqslant b, \\ & x \geqslant 0, \end{aligned} \tag{1}$$

where C is a $p \times n$ objective functions matrix. A is an $m \times n$ matrix of constraints coefficients, $b \in R^m$, and $x \in R^n$ is the vector of decision variables.

Let $X = \{x \in R^n | Ax \leqslant b\ x \geqslant 0\}$ be the feasible set in the decision space which is defined by the set of constraints. The feasible set in the objective space is $Y = CX = \{Cx | x \in X\}$. Y is also called the outcome set.

In multiobjective context, usually there is no unique optimal solution, but rather a set of efficient or non dominated solutions. So, optimality is replaced by efficiency.

Definition 1. A solution x_0 dominates another solution x_1 if and only if $Cx_0 \leqslant Cx_1$ and $Cx_0 \neq Cx_1$ i.e x_0 is no worse than x_1 according to all objectives and x_0 is strictly better than x_1 in at least one objective (minimization problem) [11].

Definition 2. A solution x_0 is said to be pareto optimal or efficient if and only if $\nexists x_1 \in X : x_1$ dominates x_0 [11].

In objective space, an efficient solution is defined similarly:

Definition 3. A solution $y_0 \in Y$ is called an efficient outcome solution if and only if $\nexists y_1 \in Y$: $y_1 \leqslant y$ and $y_1 \neq y$ [12].

Therefore, the pareto optimal set in decision space is the set of all efficient solutions X_e. And the efficient outcome set Y_e is exactly the mapping of X_e using the objectives matrix C, such that $Y_e = \{Cx | x \in X_e\}$.

In this paper, we suppose that X is a nonempty compact polyhedron. Hence, it can be easily proven that Y is also a nonempty compact polyhedron [6]. Let X_{ex} the set of all efficient vertices in decision space. Each extreme point $x \in X_{ex}$ can be mapped by C either into a single extreme point or into an non extreme point in the outcome set Y . However, for each extreme point $y \in Y_{ex}$ there exists at least one element $x = Cy$ such that $x \in X_{ex}$ [10]. One of the most important results, is that the set X_{ex} (or Y_{ex}) is connected [11]. The connectedness means that for every two efficient points, there is necessarily an efficient path between them (efficient edges relating them). This result makes a method such that simplex convenient and adaptable for exploring the efficient solutions.

Definition 4. A face is located in the boundary of the polyhedron X (resp Y). It is the intersection of X (Y) with a supporting hyperplane. A face is characterized by its dimensionality: vertices are of zero dimension and edges are one dimension, while the maximal dimension is $n - 1$ ($p - 1$) [3].

A face F is an efficient face if and only if every point on F is efficient [7]. Let $F \subset X_e$ (resp $F \subset Y_e$) be an efficient face of X (Y), F is called maximal efficient face, if there is no efficient face F_1 of higher dimension such that $F \subset F_1$ [3]. Hence, solving a multiobjective linear problem can be reduced to finding all maximal efficient faces.

The efficiency of a face can be investigated by testing a relative interior point. Thus, let F be an arbitrary convex subset of X (resp Y), and let $x \in$ the relative interior of F. If x is efficient than all the face is efficient and if x is dominated then all the relative interior set is dominated [4].