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Hamilton–Jacobi–Bellman equations and dynamic programming for power-maximizing relaxation of radiation

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Abstract

We treat simulation and power optimization of nonlinear, steady and dynamical generators of mechanical energy, in particular radiation engines. In dynamical cases, associated with downgrading of resources in time, real work is a cumulative effect obtained from a nonlinear fluid, set of engines, and an infinite bath. Dynamical state equations describe resources upgrading or downgrading in terms of temperature, work output and process controls. Recent formulae for converter's efficiency and generated power serve to derive Hamilton–Jacobi equations for the trajectory optimization. The relaxation curve of typical nonlinear system is non-exponential. Power extremization algorithms in the form of Hamilton–Jacobi–Bellman equations (HJB equations) lead to work limits and generalized availabilities. Optimal performance functions depend on end states and the problem Hamiltonian, h. As an example of limiting work from radiation, a generalized exergy flux of radiation fluid is estimated in terms of finite rates quantified by Hamiltonian h.

In many systems governing HJB equations cannot be solved analytically. Then the use of discrete counterparts of these equations and numerical methods is recommended. Algorithms of discrete dynamic programming (DP) are particularly effective as they lead directly to work limits and generalized availabilities. Convergence of these algorithms to solutions of HJB equations is discussed. A Lagrange multiplier λ helps to solve numerical algorithms of dynamic programming by eliminating the duration constraint. In analytical discrete schemes, the Legendre transformation is a significant tool leading to the original work function.
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1. Introduction

An important class of research on energy limits involves nonlinear systems driven by fluids that are restricted in their amount or flow, i.e. play role of resources. A resource is a valuable substance used in a limited amount in a practical process. Value of the resource can be quantified thermodynamically by specifying its exergy, a maximum work that can be delivered when the resource relaxes to the equilibrium. Reversible relaxation of the resource is associated with the classical exergy. When some dissipative phenomena are allowed generalized exergies are found. They include the resource availability and a minimum work lost during its production. In the classical exergy only the first property is essential.

To calculate an exergy, knowledge of a work integral is required. For thermal problems its integrand is the product of thermal efficiency and the differential of exchanged energy. Various dissipation models lead to diverse thermal efficiencies that deviate from the Carnot efficiency. In fact, generalized exergies quantify somehow these deviations.

Formally, an exergy follows from the principal function of a variational problem for extremum work under suitable boundary conditions. Other components are optimal trajectory and optimal control. In thermal systems the trajectory is characterized by temperature of the resource, $T(t)$, whereas a suitable control is Carnot temperature $T'(t)$ defined in our previous work [1,2]. Whenever $T'(t)$ differs from $T(t)$ the resource relaxes to the environment with a

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Nomenclature

A^{∞}	generalized exergy density of resource (J m^{-3})
A^{class}	density of classical exergy (J m^{-3})
a	temperature power exponent in kinetic equation (–)
a_{v}	total area of energy exchange per unit volume (m^{-1})
c_{v}	specific heat of unit volume (J m^{-3} K^{-1})
c	specific heats (J g^{-1} K^{-1}, J m^{-3} K^{-1}, J mol^{-1} K^{-1})
$D^n, \widetilde{D}^n$	generalized profit and gauge profit at stage n
DP	dynamic programming
$\mathbf{f}$	rate vector with components $f_1, \ldots, f_k, \ldots, f_s$
f_0, f_i	profit rate and process rates
G	gauge function
$\dot{G}$	resource flux (g s^{-1}, mol s^{-1})
g_1, g	partial and overall conductance (J s^{-1} K^{-a})
H	Hamiltonian function
h	Hamiltonian density in entropy units (J m^{-3} K^{-1})
l_0	Lagrangian, intensity of generalized cost
p	power output (J s^{-1})
R	minimum performance function (J, or J mol^{-1})
S, S_σ	entropy and entropy produced (J K^{-1})
T	variable temperature of resource fluid (K)
T_1, T_2	bulk temperatures of reservoirs 1 and 2 (K)
$T_{1'}$, $T_{2'}$	temperatures of circulating fluid (Fig. 3) (K)
T^n	temperature after stage n (K)
T^{e}	constant temperature of environment (K)
T'	Carnot temperature control (K)
$\dot{T} = u$	rate of control of T in non-dimensional time (K)
t	physical time (s)
$\mathbf{u}$	control vector
u and v	temperature rate controls, $\mathrm{d}T/\mathrm{d}\tau$ and $\mathrm{d}T/\mathrm{d}t$, respectively (K, K s^{-1})
V	maximum performance function (J, or J mol^{-1})
v	velocity (ms^{-1})
W and $\dot{W}$	work and power (J, J s^{-1})
w	work per unit flux of resource (J/mol)
$\mathbf{x}$	state vector
$\tilde{x}$	enlarged state vector including time
z_k	adjoint variable for kth coordinate

Greek symbols

α'	overall heat transfer coefficient (J m^{-2} s^{-1} K^{-1})
β	coefficient, frequency constant (s^{-1})
λ	Lagrange multiplier, time adjoint
$\eta = p/q_1$	first-law efficiency (–)
Φ	factor of internal irreversibility (–)
θ	time interval (s, –)
ξ	intensity index (–)
τ	non-dimensional time or number of heat transfer units (x/H_{TU}) (–)

Subscripts

k	kth state variable
m	molar flow
1,2	first and second fluid
*	modified cost or profit

Superscripts

e	environment
i	initial state
f	final state
$'$	modified quantity

finite rate and the system's efficiency deviates from that of Carnot. Only in the case when $T'(t) = T(t)$ the efficiency is Carnot, but this corresponds with an infinitely slow relaxation rate of the resource to the thermodynamic equilibrium with the environmental fluid.

The structure of this paper is as follows. Section 2 discusses various aspects of steady and dynamical optimization of power yield. Quantitative analysis of processes with resource's downgrading (in the first reservoir) and issues regarding generalization of the classical exergy for finite rates are presented in Section 3. Sections 4–6 display various Hamilton–Jacobi–Bellman (HJB) and Hamilton–Jacobi equations for extremum power production (consumption). Extensions, highlighting systems with complex kinetics (e.g. radiation) and internal dissipation are treated in Section 7. Analytical formulae for generalized exergies of some nonlinear systems are discussed in Section 8. Next, in view of severe difficulties in getting analytical solutions for systems with nonlinear kinetics discretized (difference) equations and numerical approaches are considered. Section 9 displays difference equations obtained from discretization of the continuous model of power production from the black radiation and presents the dynamic programming equation (DP equation) of the problem. Section 10 discusses convergence conditions of discrete DP schemes to solutions of continuous HJB equations. Section 11 elucidates the solving method by discrete approximations and introduces a Lagrange multiplier as a time adjoint. Section 12 shows the significance of the Legendre transform in recovering original work functions. Section 13 describes numerical procedures using dynamic programming, whereas Section 14 discusses dimensionality reduction in numerical DP algorithms. Section 15 presents most essential conclusions.

The size limitation of the present paper does not allow for inclusion of all suitable derivations to make this paper self-contained, thus the reader may need to turn to some previous works [1,2,4,5].

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