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Assessing the risk-return trade-off in loan portfolios ☆

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A R T I C L E   I N F O

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A B S T R A C T

This paper proposes a methodology to analyse the risk and return of large loan portfolios in a joint setting. I propose a tractable model to obtain the distribution of loan returns from observed interest rates and default frequencies. I follow a sectoral approach that captures the heterogeneous cyclical features of different kinds of loans and yields moments in closed form. I investigate the validity of mean–variance analysis with a value at risk constraint and study its relationship with utility maximisation. Finally, I study the efficiency of corporate and household loan portfolios in an empirical application to the Spanish banking system.

1. Introduction

The risk and return of empirical loan portfolios has generally been analysed separately. Chirinko and Guill (1991) is one of the first empirical attempts to model the credit risk associated with loan portfolios. More recently, the Basel II framework has originated the development of many quantitative models to estimate the loss distribution of loan portfolios (see Embrechts et al., 2005 for a textbook review of the literature). Meanwhile, a parallel literature studying the determinants of interest rates has simultaneously grown during the previous years (see e.g. Martín et al., 2007; Mueller, 2008). However, much less is known about the interaction between risk and profitability. Mean–variance analysis, introduced by Markowitz (1952), is the most popular tool to compare the characteristics of different assets. The theoretical banking literature has extensively used this tool to analyse the investment problem of banks (see Koehn and Santomero, 1980; Rochet, 1992 and Kim and Santomero, 1988, among others). However, these papers typically assume Gaussian returns, despite the well known asymmetric features of loan returns. More recently, Altman (1996) used the mean–variance framework to analyse the risk and return characteristics of loan and bond portfolios. However, his approach relies on ratings and market data. Unfortunately, this information is not available for loans to small and medium enterprises or households, which constitute the largest share of most banks' portfolios.

The purpose of this paper is to model the distribution of loan portfolios explicitly and study the usefulness of mean–variance analysis when the actual features of loans are taken into account. As Chamberlain (1983) shows, mean–variance analysis may still be valid in a non-Gaussian context, as long as the higher order moments are fixed once the mean and variance have been chosen. In this sense, my model generates negatively skewed loan returns but at the same time remains a function of a vector of underlying Gaussian state variables. In this context, I can argue that the means and variances of portfolio returns remain valid measures of profitability and risk, respectively. From an empirical perspective, my goal is to make this investment problem operational in broad loan portfolios, including personal loans as well as corporate loans of

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firms that have no access to the bond market. In contrast to other applications, the fact that loan portfolio returns are not observable makes the use of mean variance analysis far from trivial. Hence, returns have to be inferred from available information. Unfortunately, in most cases neither the loans themselves nor any other product from their borrowers are traded in any organised market. To overcome this problem, the usual approach is to assess credit risk by grouping loans in buckets of similar characteristics (see Foglia, 2009). Then, it is easy to compare the historical performance of loans with similar features. Following this methodology, I show how to obtain the return distribution from interest rates and historical loan default rates by loan category, which is generally the only information available to bank managers and supervisors when dealing with comprehensive samples of loans. I consider stochastic probabilities of default for each loan category, proxied by default frequencies, and model them as a probit function of an underlying multivariate Gaussian vector of state variables. This approach can be interpreted as a multiasset portfolio extension of the popular Gaussian copula of Li (2000). In this setting, I obtain closed form expressions for the expected returns, variances and covariances between different loans. The covariance matrix of returns does not only depend on the distribution of the probabilities of default, but also on the granularity of the portfolios. I also describe the multivariate distribution of returns and the value at risk (VaR) in closed form.

I analyse risk and return jointly by describing the set of feasible portfolios in mean–variance space. The frontier of this set can be easily obtained thanks to the analytical tractability of my framework, which makes mean–variance analysis in loan portfolios (almost) as easy to implement as in traded securities. The investment opportunity set may be narrowed by introducing the minimum capital requirement imposed by the regulator or possibly by an even more stringent rating target. Both conditions can be interpreted as a constraint on the minimum return that the bank must obtain, which technically corresponds to a bound on the admissible value at risk (VaR). Sentana (2003), Alexander and Baptista (2006) and Alexander et al. (2007) have previously considered mean–variance analysis with a VaR constraint when returns are elliptical. However, the negative skewness of loan portfolios can cause large biases in the elliptical estimates of the VaR. In addition, this literature does not explain how to estimate the mean and variance when returns are not observable.

I also analyse the risk-return tradeoff from a pricing point of view. In this regard, I obtain the relationship between interest rates and the risk of borrowers that ensures absence of arbitrage opportunities, which provides a useful tool to understand loan pricing in the presence of credit risk.

Finally, I consider an empirical application to Spanish loans. I use quarterly data from the Spanish credit register, from 1984.Q4 to 2008.Q4. I group corporate loans in four categories based on the economic sector of the borrower, and consider two additional groups for household loans. I propose a dynamic model to deal with the persistence and time varying features of the data. In addition, I exploit the information from banks' confidential reports about average interest rates for several classes of loans. I use this information to set the interest rates in the loan categories. In particular, I assume homogeneity within each loan class, so that all constituents from a given class have the same probability of default and interest rate. With this data, I compute the multivariate return distributions, the conditional value at risk by loan category and the mean–variance frontiers at different points of the credit risk cycle. Lastly, I investigate the validity of mean–variance analysis when bank managers maximise a constant relative risk aversion utility function.

The rest of the paper is organised as follows. I describe the model in Section 2 and introduce mean–variance analysis in Section 3. Then, I develop the pricing model in Section 4. Section 5 presents the results of the empirical application and Section 6 concludes. Proofs and auxiliary results can be found in appendices.

2. General framework

I group loans in different buckets, assuming that the components within each bucket have similar characteristics in terms of risk and price. For instance, it is possible to group corporate loans according to the economic sector of the borrower, or to arrange loans to households depending on their purpose (e.g. mortgage or consumption). However, I use a general notation in this section to accommodate much finer classifications as well.

Consider an economy with two periods: $t = 0, 1$. There is a risk-free asset, whose return is r, and K different types of loans. There are N_k loans in each of these groups, for $k = 1, \dots, K$. I denote the volume of loan i from group k as L_{ki}, while r_k is the interest rate to be paid to the lender at $t = 1$ for each loan type. Interest rates are set at $t = 0$ by the lender and accepted by the borrower. In this section, I assume that they have already been agreed and take them as given. I build on the results of this section and develop a loan pricing framework in Section 4.

I now turn to modelling borrowers' credit risk. Borrowers may default at $t = 1$, where default is driven by a binary variable D_{ki} that takes a value of 1 if i defaults and is 0 otherwise. The probability of default of any loan from group k is given by the stochastic variable π_k. In fact, π_k is not even observable at $t = 1$ unless N_k grows to infinity.[1] In case of default, the lender obtains a recovery rate δ_k, which is the proportion of the loan amount that is eventually recovered. This variable can also be stochastic. I assume that defaults are conditionally independent given π_k and δ_k for $k = 1, \dots, K$. However, notice that loans from either the same or different categories are unconditionally dependent, because the stochastic features of π_k and δ_k introduce correlation between them.

Consider the portfolio of loans held by a particular bank at time 0. Its value is the cost of the initial investment, which can be expressed as the sum of the outstanding debt at this period, i.e.

$$p = \sum_{k=1}^{K} p_k,$$

where

$$p_k = \sum_{j=1}^{N_{kt-1}} L_{jk}, \tag{1}$$

denotes the outstanding debt of the kth category of loans, for $k = 1, \dots, K$. One period later, the pay-offs generated by each class of loans can be expressed as

$$Z_k = (1 + r_k) \sum_{j=1}^{N_k} L_{jk}(1 - D_{jk}) + \sum_{j=1}^{N_k} \delta_k L_{jk} D_{jk}. \tag{2}$$

Intuitively, each borrower either repays the principal plus interest or defaults, in which case the lender only receives the recovery rate times the outstanding amount. Notice that (2) is fully consistent with the limited liability of banks (see Rochet, 1992), since the final value of the portfolio cannot be negative by construction.

Using (1) and (2), it is straightforward to write the return or yield generated by loans from group k as:

$$y_k = \frac{Z_k - p_k}{p_k} = r_k - \frac{\sum_{j=1}^{N_k} (1 + r_k - \delta_k) L_{jk} D_{jk}}{N_k \bar{L}_k}, \tag{3}$$

[1] Specifically, it can be shown that $\pi_k = \lim_{N_k \to \infty} \sum_i D_{ik}/N_k$ under standard regularity conditions. I exploit this feature in the empirical application.